

Evaluating Limits Worksheet

Evaluate the following limits without using a calculator.

1) $\lim_{x \rightarrow 3} \frac{2x^2 - 5x - 3}{x - 3}$

$$= \lim_{x \rightarrow 3} \frac{(2x+1)(x-3)}{(x-3)}$$

$$= \lim_{x \rightarrow 3} 2x+1 = 2(3)+1 = \boxed{7}$$

(one method)

factor $2x^2 - 5x - 3$

$$2x^2 + 1x - 6x - 3$$

$$x(2x+1) - 3(2x+1)$$

$$(2x+1)(x-3)$$

2) $\lim_{x \rightarrow 2} \frac{x^4 - 16}{x - 2}$

$$= \lim_{x \rightarrow 2} \frac{(x^2+4)(x+2)(x-2)}{(x-2)}$$

$$= \lim_{x \rightarrow 2} (x^2+4)(x+2) = (2^2+4)(2+2) = 8 \cdot 4 = \boxed{32}$$

factor $x^4 - 16$

$$(x^2+4)(x^2-4)$$

$$(x^2+4)(x+2)(x-2)$$

3) $\lim_{x \rightarrow -1} \frac{x^4 + 3x^3 - x^2 + x + 4}{x + 1}$

$$= \lim_{x \rightarrow -1} \frac{(x+1)(x^3 + 2x^2 - 3x + 4)}{x+1}$$

$$= \lim_{x \rightarrow -1} x^3 + 2x^2 - 3x + 4 = -1 + 2 + 3 + 4 = \boxed{8}$$

polynomial long division

$$\begin{array}{r} x^3 + 2x^2 - 3x + 4 \\ x+1 \overline{) x^4 + 3x^3 - x^2 + x + 4} \end{array}$$

$$-(x^4 + x^3)$$

$$2x^2 - x^2$$

$$-(2x^2 + 2x)$$

$$-3x^2 + x$$

$$-(-3x^2 - 3x)$$

$$4x + 4$$

$$-(4x + 4)$$

0 ✓

4) $\lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x}$

$$= \lim_{x \rightarrow 0} \frac{(\sqrt{x+4} - 2)(\sqrt{x+4} + 2)}{x(\sqrt{x+4} + 2)}$$

$$= \lim_{x \rightarrow 0} \frac{x+4-4}{x(\sqrt{x+4} + 2)}$$

$$\lim_{x \rightarrow 0} \frac{x}{x(\sqrt{x+4} + 2)}$$

$$= \lim_{x \rightarrow 0} \frac{1}{\sqrt{x+4} + 2}$$

$$= \frac{1}{\sqrt{4} + 2} = \boxed{\frac{1}{4}}$$

5) $\lim_{x \rightarrow 3} \frac{\sqrt{x+6} - x}{x - 3}$

$$= \lim_{x \rightarrow 3} \frac{(\sqrt{x+6} - x)(\sqrt{x+6} + x)}{(x-3)(\sqrt{x+6} + x)} = \lim_{x \rightarrow 3} \frac{x+6-x^2}{(x-3)(\sqrt{x+6} + x)} = \lim_{x \rightarrow 3} \frac{-(x^2-x-6)}{(x-3)(\sqrt{x+6} + x)} = \lim_{x \rightarrow 3} \frac{-(x-3)(x+2)}{(x-3)(\sqrt{x+6} + x)}$$

$$= \lim_{x \rightarrow 3} \frac{-(x+2)}{\sqrt{x+6} + x} = \frac{-(3+2)}{\sqrt{3+6} + 3} = \boxed{\frac{-5}{6}}$$

$$6) \lim_{x \rightarrow -2} \frac{\frac{1}{2} + \frac{1}{x}}{x+2} = \lim_{x \rightarrow -2} \frac{\frac{x+2}{2x}}{\frac{x+2}{1}} = \lim_{x \rightarrow -2} \frac{x+2}{2x} \cdot \frac{1}{x+2} = \lim_{x \rightarrow -2} \frac{1}{2x} = \frac{1}{2(-2)} = \boxed{-\frac{1}{4}}$$

$$7) \lim_{x \rightarrow \frac{1}{2}} \frac{x^{-1} - 2}{x - \frac{1}{2}} = \lim_{x \rightarrow \frac{1}{2}} \frac{\frac{1}{x} - \frac{2}{1}}{\frac{x}{1} - \frac{1}{2}} = \lim_{x \rightarrow \frac{1}{2}} \frac{\frac{1-2x}{x}}{\frac{2x-1}{2}} = \lim_{x \rightarrow \frac{1}{2}} \frac{1-2x}{x} \cdot \frac{2}{2x-1} =$$

$$= \lim_{x \rightarrow \frac{1}{2}} \frac{-(2x-1) \cdot 2}{x(2x-1)} = \lim_{x \rightarrow \frac{1}{2}} \frac{-2}{x} = \frac{-2}{\frac{1}{2}} = -2 \cdot \frac{2}{1} = \boxed{-4}$$

$$8) \lim_{x \rightarrow 3} \frac{\frac{1}{x^2} - \frac{1}{9}}{x-3} = \lim_{x \rightarrow 3} \frac{\frac{9-x^2}{9x^2}}{\frac{x-3}{1}} = \lim_{x \rightarrow 3} \frac{9-x^2}{9x^2} \cdot \frac{1}{x-3} = \lim_{x \rightarrow 3} \frac{-(x^2-9)}{9x^2} \cdot \frac{1}{x-3} = \lim_{x \rightarrow 3} \frac{-(x+3)(x-3)}{9x^2(x-3)}$$

$$= \lim_{x \rightarrow 3} \frac{-(x+3)}{9x^2} = \frac{-6}{9(9)} = \frac{-6}{81} = \boxed{-\frac{2}{27}}$$

$$9) \lim_{x \rightarrow 0} \frac{|x+2| - 2}{|x|}$$

because

$$\lim_{x \rightarrow 0^+} \neq \lim_{x \rightarrow 0^-}$$

$$\lim_{x \rightarrow 0} \frac{|x+2| - 2}{x} \text{ is } \boxed{\text{undefined}}$$

$$10) \lim_{x \rightarrow 3} \frac{|x^2 - 9|}{|x - 3|}$$

$$\lim_{x \rightarrow 0^+} \frac{|x+2| - 2}{|x|}$$

$$\lim_{x \rightarrow 0^-} \frac{|x+2| - 2}{|x|}$$

$$= \lim_{x \rightarrow 0^+} \frac{x+2-2}{x}$$

$$= \lim_{x \rightarrow 0^-} \frac{x+2-2}{-x}$$

$$= \lim_{x \rightarrow 0^+} \frac{x}{x} = \lim_{x \rightarrow 0^+} 1 = \boxed{1}$$

$$= \lim_{x \rightarrow 0^-} \frac{x}{-x} = \lim_{x \rightarrow 0^-} -1 = \boxed{-1}$$

$$\lim_{x \rightarrow 3^+} \frac{|x^2 - 9|}{|x - 3|} = \lim_{x \rightarrow 3^+} \frac{x^2 - 9}{x - 3} = \lim_{x \rightarrow 3^+} \frac{(x+3)(x-3)}{x-3} = \lim_{x \rightarrow 3^+} x+3 = 3+3 = 6$$

$$\lim_{x \rightarrow 3^-} \frac{|x^2 - 9|}{|x - 3|} = \lim_{x \rightarrow 3^-} \frac{-(x^2 - 9)}{-(x-3)} = \lim_{x \rightarrow 3^-} \frac{-(x+3)(x-3)}{-(x-3)} = \lim_{x \rightarrow 3^-} x+3 = 3+3 = 6$$

$$\left. \begin{array}{l} \text{so } \lim_{x \rightarrow 3} \frac{|x^2 - 9|}{|x - 3|} \\ = \boxed{6} \end{array} \right\}$$

$$11) \lim_{x \rightarrow 0} \left(\frac{5}{x^2 + x} - \frac{5}{x} \right)$$

$$= \lim_{x \rightarrow 0} \frac{5}{x(x+1)} - \frac{5(x+1)}{x(x+1)} = \lim_{x \rightarrow 0} \frac{5 - 5(x+1)}{x(x+1)} = \lim_{x \rightarrow 0} \frac{5 - 5x - 5}{x(x+1)} = \lim_{x \rightarrow 0} \frac{-5x}{x(x+1)} = \lim_{x \rightarrow 0} \frac{-5}{x+1} = \boxed{-5}$$