Limits: Numeric Solutions

Now that you know how to solve a limit graphically, you may be asking yourself: 'That's great, but what about when there isn't a graph in the problem?' That is a good question, and that is what this next section is about. There are a many better (and more accurate) ways to find the value of the limit than graphing or plugging in numbers that get closer and closer to the value of interest. These solution methods fall under three categories: substitution, factoring, and the conjugate method. But first things first, let's discuss some of the general rules for limits.

Limit Rules

Here are some of the general limit rules (with $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = M$):

1. **Sum Rule**: The limit of the sum of two functions is the sum of their limits
$$\lim_{x \to c}\big(f(x) + g(x)\big) = \lim_{x \to c} f(x) + \lim_{x \to c} g(x) = L + M$$
2. **Difference Rule**: The limit of the difference of two functions is the difference of their limits
$$\lim_{x \to c}\big(f(x) - g(x)\big) = \lim_{x \to c} f(x) - \lim_{x \to c} g(x) = L - M$$
3. **Product Rule**: The limit of a product of two functions is the product of their limits
$$\lim_{x \to c}\big(f(x) * g(x)\big) = \lim_{x \to c} f(x) * \lim_{x \to c} g(x) = L * M$$
4. **Constant Multiple Rule**: The limits of a constant times a function is the constant times the limit of the function
$$\lim_{x \to c}\big(k * f(x)\big) = k \lim_{x \to c} f(x) = k * L$$
5. **Quotient Rule**: The limit of a quotient of two functions is the quotient of their limits, provided the limit of the denominator is not zero
$$\lim_{x \to c}\left(\frac{f(x)}{g(x)}\right) = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)} = \frac{L}{M}, \quad M \neq 0$$

Limit Rule Examples Find the following limits using the above limit rules:

1. $\lim_{x\to c}(x^3 + 4x^2 - 3) = \lim_{x\to c}(x^3) + \lim_{x\to c}(4x^2) - \lim_{x\to c}(3)$
   $= c^3 + 4c^2 - 3$

2. $\lim_{x\to c} \frac{x^4+x^2-1}{x^2+5} = \frac{\lim_{x\to c}(x^4+x^2-1)}{\lim_{x\to c}(x^2+5)}$
   $$= \frac{\lim\limits_{x\to c}(x^4) + \lim\limits_{x\to c}(x^2) - \lim\limits_{x\to c}(1)}{\lim\limits_{x\to c}(x^2) + \lim\limits_{x\to c}(5)}$$
   $$= \frac{c^4 + c^2 - 1}{c^2 + 5}$$

3. $\lim_{x\to 1} \frac{x^4+x^2-1}{x^2+5} = \frac{\lim_{x\to 1}(x^4+x^2-1)}{\lim_{x\to 1}(x^2+5)}$
   $$= \frac{\lim\limits_{x\to 1}(x^4) + \lim\limits_{x\to 1}(x^2) - \lim\limits_{x\to 1}(1)}{\lim\limits_{x\to 1}(x^2) + \lim\limits_{x\to 1}(5)}$$
   $$= \frac{1^4 + 1^2 - 1}{1^2 + 5} = \frac{1}{6}$$