Factoring Method

Consider the function $(x) = \frac{x^2-9}{x+3}$. How would you find the limit of f as x approaches -3? If you try to use substitution to find the limit, world-ending paradoxes ensue:

$$\lim_{x \to -3} \frac{x^2 - 9}{x + 3} = \frac{(-3)^2 - 9}{(-3) + 3} = \frac{0}{0}$$

But fear not, this answer just tells us that we must use a different method to find the limit, because the function likely has a "hole" at the given *x* value. Therefore, the factoring method can be tried. To start this method, the numerator and denominator must be factored (in this case the denominator is "factored" already).

$$\lim_{x \to -3} \frac{(x + 3)(x - 3)}{x + 3}$$

The factor $(x + 3)$ can be canceled to get the much simpler limit expression of $\lim_{x \to -3}(x - 3)$ that can easily be evaluated via substitution:

$$\lim_{x \to -3}(x - 3) = -3 - 3 = -6$$

Therefore, the result of the limit can be found, with the understanding that there is a "hole" in the graph at $x = -3$. Hence, $\lim_{x \to -3} \frac{x^2-9}{x+3} = -6$.