Conjugate Method

The conjugate of a binomial expression (i.e. an expression with two terms, *you can tell this because of the Latin root bi- meaning two*) is the same expression with opposite middle signs. For example, the conjugate of $(\sqrt{x} - 5)$ is $(\sqrt{x} + 5)$. This is really useful if you have a radical in your limit. This is because the product of two conjugates containing radicals will, itself, contain no radical expressions. See below:

$$(\sqrt{x} - 5)(\sqrt{x} + 5) = \sqrt{x^2} + 5\sqrt{x} - 5\sqrt{x} - 25 = x - 25$$

You should use the conjugate method whenever you have a limit problem containing radicals for which substitution does not work.

Example:

Evaluate $\lim_{x \to 5} \frac{\sqrt{x+11}-4}{x-5}$

First try the substitution method:

$$\lim_{x \to 5} \frac{\sqrt{x+11} - 4}{x - 5} = \frac{\sqrt{5+11} - 4}{5 - 5} = \frac{0}{0}$$

Well, another hole in the universe, or at least the graph. Indicating that you'll need another method to find the limit since the function probably has a hole at $x = 5$. To start, multiply both the numerator and denominator by the conjugate of the radical expression $(\sqrt{x+11} + 4)$:

$$\lim_{x \to 5} \frac{\sqrt{x+11} - 4}{x - 5} * \frac{(\sqrt{x+11} + 4)}{(\sqrt{x+11} + 4)}$$

$$\lim_{x \to 5} \frac{(x + 11) - 16}{(x - 5)(\sqrt{x+11} + 4)}$$

$$\lim_{x \to 5} \frac{x - 5}{(x - 5)(\sqrt{x+11} + 4)}$$

Cancel the $x - 5$ factor in the numerator and denominator.

$$\lim_{x \to 5} \frac{1}{(\sqrt{x+11} + 4)} = \frac{1}{(\sqrt{5+11} + 4)} = \frac{1}{8}$$