Evaluating Limits by Factoring

1. Evaluate this limit.

$$\lim_{x \to 9} \frac{x - 9}{x^2 - 81}$$

2. Evaluate this limit.

$$\lim_{x \to 2} \frac{x^2 - 3x + 2}{x^2 - 5x + 6}$$

3. Evaluate this limit.

$$\lim_{x \to -4} \frac{x + 4}{4x + 16}$$

4. Evaluate this limit.

$$\lim_{x \to -2} \frac{x+2}{x^2+x-2}$$

5. Evaluate this limit.

$$\lim_{x \to 1} \frac{35x-35}{40x-40}$$

6. Evaluate this limit.

$$\lim_{x \to -1} \frac{x^2+6x+5}{x+1}$$

7. Evaluate this limit.

$$\lim_{x \to 7} \frac{-x^2 + 16x - 63}{x^2 - 2x - 35}$$

8. Evaluate this limit.

$$\lim_{x \to -2} \frac{x + 2}{x^2 + 6x + 8}$$

9. Evaluate this limit.

$$\lim_{x \to 0} \frac{2x^2 + 10x}{3x^2 + 15x}$$

10. Evaluate this limit.

$$\lim_{x \to -7} \frac{x + 7}{x^2 + 4x - 21}$$

11. Evaluate this limit.

$$\lim_{x \to -2} \frac{x^2 - 3x - 10}{x^2 + x - 2}$$

12. Evaluate this limit.

$$\lim_{x \to 8} \frac{x - 8}{x^2 - 2x - 48}$$

13. Evaluate this limit.

$$\lim_{x \to 6} \frac{6 - x}{3x - 18}$$

14. Evaluate this limit.

$$\lim_{x \to 4} \frac{28 - 7x}{x - 4}$$

15. Evaluate this limit.

$$\lim_{x \to 7} \frac{4x - 28}{5x^3 - 35x^2}$$

Answers ... Problem 1

1. Evaluate this limit.

$$\lim_{x\to 9}\frac{x-9}{x^2-81}$$

Solution:

First, attempt to evaluate the limit using direct substitution. Substitute 9 into the limit for x.

$$\lim_{x\to 9}\frac{x-9}{x^2-81}=\frac{9-9}{9^2-81}=\frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\lim_{x\to 9}\frac{x-9}{x^2-81}=\lim_{x\to 9}\frac{x-9}{(x+9)(x-9)}=\lim_{x\to 9}\frac{1}{x+9}$$

Substitute 9 into the limit for x.

$$\lim_{x\to 9}\frac{1}{x+9}=\frac{1}{9+9}=\frac{1}{18}$$

Answer: $\lim_{x\to 9}\frac{x-9}{x^2-81}=\frac{1}{18}$

Answers ... Problem 2

2. Evaluate this limit.

$$\lim_{x\to 2}\frac{x^2-3x+2}{x^2-5x+6}$$

Solution:

First, attempt to evaluate the limit using direct substitution. Substitute 2 into the limit for x.

$$\lim_{x\to 2}\frac{x^2-3x+2}{x^2-5x+6}=\frac{(2)^2-3(2)+2}{(2)^2-5(2)+6}=\frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\lim_{x\to 2}\frac{x^2-3x+2}{x^2-5x+6}=\lim_{x\to 2}\frac{(x-2)(x-1)}{(x-3)(x-2)}=\lim_{x\to 2}\frac{x-1}{x-3}$$

Substitute 2 into the limit for x.

$$\lim_{x\to 2}\frac{x-1}{x-3}=\frac{2-1}{2-3}=\frac{1}{-1}=-1$$

Answer: $\lim_{x\to 2}\frac{x^2-3x+2}{x^2-5x+6}=-1$

Answers ... Problem 3

3. Evaluate this limit.

$$\lim_{x\to -4} \frac{x+4}{4x+16}$$

Solution:

First, attempt to evaluate the limit using direct substitution. Substitute -4 into the limit for x.

$$\lim_{x\to -4} \frac{x+4}{4x+16} = \frac{-4+4}{4(-4)+16} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\lim_{x\to -4} \frac{x+4}{4x+16} = \lim_{x\to -4} \frac{x+4}{4(x+4)} = \lim_{x\to -4} \frac{1}{4} = \frac{1}{4}$$

Answer: $\lim_{x\to -4} \frac{x+4}{4x+16} = \frac{1}{4}$

Answers ... Problem 4

4. Evaluate this limit.

$$\lim_{x\to -2} \frac{x+2}{x^2+x-2}$$

Solution:

First, attempt to evaluate the limit using direct substitution. Substitute -2 into the limit for x.

$$\lim_{x\to -2} \frac{x+2}{x^2+x-2} = \frac{-2+2}{(-2)^2+(-2)-2} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\lim_{x\to -2} \frac{x+2}{x^2+x-2} = \lim_{x\to -2} \frac{x+2}{(x+2)(x-1)} = \lim_{x\to -2} \frac{1}{x-1}$$

Substitute -2 into the limit for x.

$$\lim_{x\to -2} \frac{1}{x-1} = \frac{1}{-2-1} = \frac{1}{-3} = -\frac{1}{3}$$

Answer: $\lim_{x\to -2} \frac{x+2}{x^2+x-2} = -\frac{1}{3}$

Answers ... Problem 5

5. Evaluate this limit.

$$\lim_{x\to 1}\frac{35x-35}{40x-40}$$

Solution:

First, attempt to evaluate the limit using direct substitution. Substitute 1 into the limit for x.

$$\lim_{x\to 1}\frac{35x-35}{40x-40}=\frac{35(1)-35}{40(1)-40}=\frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\lim_{x\to 1}\frac{35x-35}{40x-40}=\lim_{x\to 1}\frac{35(x-1)}{40(x-1)}=\lim_{x\to 1}\frac{35}{40}=\frac{7}{8}$$

Answer: $\lim_{x\to 1}\frac{35x-35}{40x-40}=\frac{7}{8}$

Answers ... Problem 6

6. Evaluate this limit.

$$\lim_{x\to -1} \frac{x^2+6x+5}{x+1}$$

Solution:

First, attempt to evaluate the limit using direct substitution. Substitute -1 into the limit for x.

$$\lim_{x\to -1} \frac{x^2+6x+5}{x+1} = \frac{(-1)^2+6(-1)+5}{-1+1} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simpli the limit.

$$\lim_{x\to -1} \frac{x^2+6x+5}{x+1} = \lim_{x\to -1} \frac{(x+1)(x+5)}{x+1} = \lim_{x\to -1} (x+5)$$

Substitute -1 into the limit for x.

$$\lim_{x\to -1} (x+5) = -1+5 = 4$$

Answer: $\lim_{x\to -1} \frac{x^2+6x+5}{x+1} = 4$

Answers ... Problem 7

7. Evaluate this limit.

$$\lim_{x\to 7}\frac{-x^2+16x-63}{x^2-2x-35}$$

Solution:

First, attempt to evaluate the limit using direct substitution. Substitute 7 into the limit for x.

$$\lim_{x\to 7}\frac{-x^2+16x-63}{x^2-2x-35}=\frac{-7^2+16(7)-63}{7^2-2(7)-35}=\frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\lim_{x\to 7}\frac{-x^2+16x-63}{x^2-2x-35}=\lim_{x\to 7}\frac{-(x^2-16x+63)}{x^2-2x-35}$$

$$\lim_{x\to 7}\frac{-(x^2-16x+63)}{x^2-2x-35}=\lim_{x\to 7}\frac{-(x-7)(x-9)}{(x+5)(x-7)}$$

$$\lim_{x\to 7}\frac{-(x-9)}{x+5}$$

Substitute 7 into the limit for x.

$$\lim_{x\to 7}\frac{-(x-9)}{x+5}=\frac{-(7-9)}{7+5}=\frac{2}{12}=\frac{1}{6}$$

Answer: $\lim_{x\to 7}\frac{-x^2+16x-63}{x^2-2x-35}=\frac{1}{6}$

Answers ... Problem 8

8. Evaluate this limit.

$$\lim_{x\to -2} \frac{x+2}{x^2+6x+8}$$

Solution:

First, attempt to evaluate the limit using direct substitution. Substitute -2 into the limit for x.

$$\lim_{x\to -2} \frac{x+2}{x^2+6x+8} = \frac{-2+2}{(-2)^2+6(-2)+8} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\lim_{x\to -2} \frac{x+2}{x^2+6x+8} = \lim_{x\to -2} \frac{x+2}{(x+2)(x+4)} = \lim_{x\to -2} \frac{1}{x+4}$$

Substitute -2 into the limit for x.

$$\lim_{x\to -2} \frac{1}{x+4} = \frac{1}{-2+4} = \frac{1}{2}$$

Answer: $\lim_{x\to -2} \frac{x+2}{x^2+6x+8} = \frac{1}{2}$

Answers ... Problem 9

9. Evaluate this limit.

$$\lim_{x\to 0}\frac{2x^2+10x}{3x^2+15x}$$

Solution:

First, attempt to evaluate the limit using direct substitution. Substitute 0 into the limit for x.

$$\lim_{x\to 0}\frac{2x^2+10x}{3x^2+15x}=\frac{2(0)^2+10(0)}{3(0)^2+15(0)}=\frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\lim_{x\to 0}\frac{2x^2+10x}{3x^2+15x}=\lim_{x\to 0}\frac{2x(x+5)}{3x(x+5)}=\frac{2}{3}$$

Answer: $\lim_{x\to 0}\frac{2x^2+10x}{3x^2+15x}=\frac{2}{3}$

Answers ... Problem 10

10. Evaluate this limit.

$$\lim_{x\to -7} \frac{x+7}{x^2+4x-21}$$

Solution:

First, attempt to evaluate the limit using direct substitution. Substitute -7 into the limit for x.

$$\lim_{x\to -7} \frac{x+7}{x^2+4x-21} = \frac{-7+7}{(-7)^2+4(-7)-21} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\lim_{x\to -7} \frac{x+7}{x^2+4x-21} = \lim_{x\to -7} \frac{x+7}{(x+7)(x-3)} = \lim_{x\to -7} \frac{1}{x-3}$$

Substitute -7 into the limit for x.

$$\lim_{x\to -7} \frac{1}{x-3} = \frac{1}{-7-3} = -\frac{1}{10}$$

Answer: $\lim_{x\to -7} \frac{x+7}{x^2+4x-21} = -\frac{1}{10}$

Answers ... Problem 11

11. Evaluate this limit.

$$\lim_{x\to -2}\frac{x^2-3x-10}{x^2+x-2}$$

Solution:

First, attempt to evaluate the limit using direct substitution. Substitute -2 into the limit for x.

$$\lim_{x\to -2}\frac{x^2-3x-10}{x^2+x-2}=\frac{(-2)^2-3(-2)-10}{(-2)^2+(-2)-2}=\frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\lim_{x\to -2}\frac{x^2-3x-10}{x^2+x-2}=\lim_{x\to -2}\frac{(x-5)(x+2)}{(x+2)(x-1)}=\lim_{x\to -2}\frac{x-5}{x-1}$$

Substitute -2 into the limit for x.

$$\lim_{x\to -2}\frac{x-5}{x-1}=\frac{-2-5}{-2-1}=\frac{-7}{-3}=\frac{7}{3}$$

Answer: $\lim_{x\to -2}\frac{x^2-3x-10}{x^2+x-2}=\frac{7}{3}$

Answers ... Problem 12

12. Evaluate this limit.

$$\lim_{x\to 8}\frac{x-8}{x^2-2x-48}$$

Solution:

First, attempt to evaluate the limit using direct substitution. Substitute 8 into the limit for x.

$$\lim_{x\to 8}\frac{x-8}{x^2-2x-48}=\frac{8-8}{8^2-2(8)-48}=\frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\lim_{x\to 8}\frac{x-8}{x^2-2x-48}=\lim_{x\to 8}\frac{x-8}{(x+6)(x-8)}=\lim_{x\to 8}\frac{1}{x+6}$$

Substitute 8 into the limit for x.

$$\lim_{x\to 8}\frac{1}{x+6}=\frac{1}{8+6}=\frac{1}{14}$$

Answer: $\lim_{x\to 8}\frac{x-8}{x^2-2x-48}=\frac{1}{14}$

Answers ... Problem 13

13. Evaluate this limit.

$$\lim_{x\to 6}\frac{6-x}{3x-18}$$

Solution:

First, attempt to evaluate the limit using direct substitution. Substitute 6 into the limit for x.

$$\lim_{x\to 6}\frac{6-x}{3x-18}=\frac{6-6}{3(6)-18}=\frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\lim_{x\to 6}\frac{6-x}{3x-18}=\lim_{x\to 6}\frac{-(x-6)}{3(x-6)}=\lim_{x\to 6}\frac{-1}{3}=-\frac{1}{3}$$

Answer: $\lim_{x\to 6}\frac{6-x}{3x-18}=-\frac{1}{3}$

Answers ... Problem 14

14. Evaluate this limit.

$$\lim_{x\to 4}\frac{28-7x}{x-4}$$

Solution:

First, attempt to evaluate the limit using direct substitution. Substitute 4 into the limit for x.

$$\lim_{x\to 4}\frac{28-7x}{x-4}=\frac{28-7(4)}{4-4}=\frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\lim_{x\to 4}\frac{28-7x}{x-4}=\lim_{x\to 4}\frac{-7(x-4)}{x-4}=\lim_{x\to 4}\frac{-7}{1}=-7$$

Answer: $\lim_{x\to 4}\frac{28-7x}{x-4}=-7$

Answers ... Problem 15

15. Evaluate this limit.

$$\lim_{x\to 7}\frac{4x-28}{5x^3-35x^2}$$

Solution:

First, attempt to evaluate the limit using direct substitution. Substitute 7 into the limit for x.

$$\lim_{x\to 7}\frac{4x-28}{5x^3-35x^2}=\frac{4(7)-28}{5(7)^3-35(7)^2}=\frac{28-28}{1715-1715}=\frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\lim_{x\to 7}\frac{4x-28}{5x^3-35x^2}=\lim_{x\to 7}\frac{4(x-7)}{5x^2(x-7)}=\lim_{x\to 7}\frac{4}{5x^2}$$

Substitute 7 into the limit for x.

$$\lim_{x\to 7}\frac{4}{5x^2}=\frac{4}{5(7)^2}=\frac{4}{245}$$

Answer: $\lim_{x\to 7}\frac{4x-28}{5x^3-35x^2}=\frac{4}{245}$