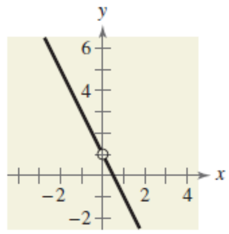
Limits for Practice

In Exercises 1–4, use the graph to determine each limit visually (if it exists). Then identify another function that agrees with the given function at all but one point.

1. $g(x) = \dfrac{-2x^2 + x}{x}$

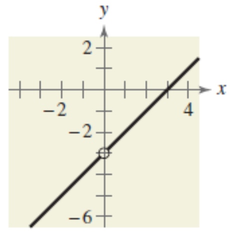


(a) $\lim\limits_{x \to 0} g(x)$

(b) $\lim\limits_{x \to -1} g(x)$

(c) $\lim\limits_{x \to -2} g(x)$

2. $h(x) = \dfrac{x^2 - 3x}{x}$

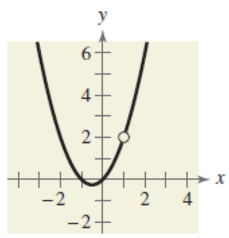


(a) $\lim\limits_{x \to -2} h(x)$

(b) $\lim\limits_{x \to 0} h(x)$

(c) $\lim\limits_{x \to 3} h(x)$

3. $g(x) = \dfrac{x^3 - x}{x - 1}$

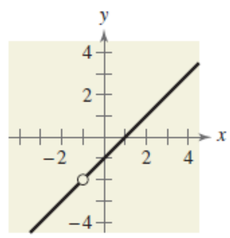


(a) $\lim\limits_{x \to 1} g(x)$

(b) $\lim\limits_{x \to -1} g(x)$

(c) $\lim\limits_{x \to 0} g(x)$

4. $f(x) = \dfrac{x^2 - 1}{x + 1}$



(a) $\lim\limits_{x \to 1} f(x)$

(b) $\lim\limits_{x \to 2} f(x)$

(c) $\lim\limits_{x \to -1} f(x)$

In Exercises 5–26, find the limit (if it exists). Use a graphing utility to verify your result graphically.

5. $\lim\limits_{x \to 6} \dfrac{x - 6}{x^2 - 36}$

6. $\lim\limits_{x \to 5} \dfrac{5 - x}{x^2 - 25}$

7. $\lim\limits_{x \to -1} \dfrac{1 - 2x - 3x^2}{1 + x}$

8. $\lim\limits_{x \to -2} \dfrac{2x^2 + 3x - 2}{x + 2}$

9. $\lim\limits_{t \to 2} \dfrac{t^3 - 8}{t - 2}$

10. $\lim\limits_{t \to -3} \dfrac{t^3 + 27}{t + 3}$

11. $\lim\limits_{x \to 2} \dfrac{x^5 - 32}{x - 2}$

12. $\lim\limits_{x \to 1} \dfrac{x^4 - 1}{x - 1}$

13. $\lim\limits_{y \to 0} \dfrac{\sqrt{5 + y} - \sqrt{5}}{y}$

14. $\lim\limits_{z \to 0} \dfrac{\sqrt{7 - z} - \sqrt{7}}{z}$

15. $\lim\limits_{x \to 0} \dfrac{\sqrt{x + 3} - \sqrt{3}}{x}$

16. $\lim\limits_{x \to 0} \dfrac{\sqrt{x + 4} - 2}{x}$

17. $\lim\limits_{x \to 0} \dfrac{\sqrt{2x + 1} - 1}{x}$

18. $\lim\limits_{x \to 9} \dfrac{3 - \sqrt{x}}{x - 9}$

19. $\lim\limits_{x \to -3} \dfrac{\sqrt{x + 7} - 2}{x + 3}$

20. $\lim\limits_{x \to 2} \dfrac{4 - \sqrt{18 - x}}{x - 2}$

21. $\lim\limits_{x \to 0} \dfrac{\dfrac{1}{x + 1} - 1}{x}$

22. $\lim\limits_{x \to 0} \dfrac{\dfrac{1}{4 - x} - \dfrac{1}{4}}{x}$

23. $\lim\limits_{x \to 0} \dfrac{\dfrac{1}{x + 4} - \dfrac{1}{4}}{x}$

24. $\lim\limits_{x \to 0} \dfrac{\dfrac{1}{2 + x} - \dfrac{1}{2}}{x}$

25. $\lim\limits_{x \to 0} \dfrac{\sec x}{\tan x}$

26. $\lim\limits_{x \to \pi/2} \dfrac{1 - \sin x}{\cos x}$

Limits for Practice

In Exercises 27–38, use a graphing utility to graph the function and approximate the limit accurate to three decimal places.

27. $\lim_{x\to 0} \frac{e^{2x}-1}{x}$

28. $\lim_{x\to 0} \frac{1-e^{-x}}{x}$

29. $\lim_{x\to 0^+} (x \ln x)$

30. $\lim_{x\to 0^+} (x^2 \ln x)$

31. $\lim_{x\to 0} \frac{\sin 2x}{x}$

32. $\lim_{x\to 0} \frac{\sin 3x}{x}$

33. $\lim_{x\to 0} \frac{\tan x}{x}$

34. $\lim_{x\to 0} \frac{1-\cos 2x}{x}$

35. $\lim_{x\to 1} \frac{1-\sqrt[3]{x}}{1-x}$

36. $\lim_{x\to 1} \frac{\sqrt[3]{x}-x}{x-1}$

37. $\lim_{x\to 0} (1-x)^{2/x}$

38. $\lim_{x\to 0} (1+2x)^{1/x}$

Graphical, Numerical, and Algebraic Analysis In Exercises 39–42, (a) graphically approximate the limit (if it exists) by using a graphing utility to graph the function, (b) numerically approximate the limit (if it exists) by using the *table* feature of a graphing utility to create a table, and (c) algebraically evaluate the limit (if it exists) by the appropriate technique(s).

39. $\lim_{x\to 1^-} \frac{x-1}{x^2-1}$

40. $\lim_{x\to 5^+} \frac{5-x}{25-x^2}$

41. $\lim_{x\to 16^+} \frac{4-\sqrt{x}}{x-16}$

42. $\lim_{x\to 0^-} \frac{\sqrt{x+2}-\sqrt{2}}{x}$

In Exercises 43–50, graph the function. Determine the limit (if it exists) by evaluating the corresponding one-sided limits.

43. $\lim_{x\to 6} \frac{|x-6|}{x-6}$

44. $\lim_{x\to 2} \frac{|x-2|}{x-2}$

45. $\lim_{x\to 1} \frac{1}{x^2+1}$

46. $\lim_{x\to 1} \frac{1}{x^2-1}$

47. $\lim_{x\to 2} f(x)$ where $f(x) = \begin{cases} x-1, & x \le 2 \\ 2x-3, & x > 2 \end{cases}$

48. $\lim_{x\to 1} f(x)$ where $f(x) = \begin{cases} 2x+1, & x < 1 \\ 4-x^2, & x \ge 1 \end{cases}$

49. $\lim_{x\to 1} f(x)$ where $f(x) = \begin{cases} 4-x^2, & x \le 1 \\ 3-x, & x > 1 \end{cases}$

50. $\lim_{x\to 0} f(x)$ where $f(x) = \begin{cases} 4-x^2, & x \le 0 \\ x+4, & x > 0 \end{cases}$

In Exercises 51–56, use a graphing utility to graph the function and the equations $y = x$ and $y = -x$ in the same viewing window. Use the graph to find $\lim_{x\to 0} f(x)$.

51. $f(x) = x \cos x$

52. $f(x) = |x \sin x|$

53. $f(x) = |x| \sin x$

54. $f(x) = |x| \cos x$

55. $f(x) = x \sin \frac{1}{x}$

56. $f(x) = x \cos \frac{1}{x}$

In Exercises 57 and 58, state which limit can be evaluated by using direct substitution. Then evaluate or approximate each limit.

57. (a) $\lim_{x\to 0} x^2 \sin x^2$ (b) $\lim_{x\to 0} \frac{\sin x^2}{x^2}$

58. (a) $\lim_{x\to 0} \frac{x}{\cos x}$ (b) $\lim_{x\to 0} \frac{1-\cos x}{x}$

In Exercises 59–66, find $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$.

59. $f(x) = 3x - 1$

60. $f(x) = 5 - 6x$

61. $f(x) = \sqrt{x}$

62. $f(x) = \sqrt{x-2}$

63. $f(x) = x^2 - 3x$

64. $f(x) = 4 - 2x - x^2$

65. $f(x) = \frac{1}{x+2}$

66. $f(x) = \frac{1}{x-1}$

Free-Falling Object In Exercises 67 and 68, use the position function $s(t) = -16t^2 + 128$, which gives the height (in feet) of a free-falling object. The velocity at time $t = a$ seconds is given by $\lim_{t\to a} [s(a) - s(t)]/(a - t)$.

67. Find the velocity when $t = 1$ second.

68. Find the velocity when $t = 2$ seconds.

69. *Salary Contract* A union contract guarantees a 10% salary increase yearly for 3 years. For a current salary of \$28,000, the salary $f(t)$ (in thousands of dollars) for the next 3 years is given by

$$f(t) = \begin{cases} 28.00, & 0 < t \le 1 \\ 30.80, & 1 < t \le 2 \\ 33.88, & 2 < t \le 3 \end{cases}$$

where t represents the time in years. Show that the limit of f as $t \to 2$ does not exist.

70. *Consumer Awareness* The cost of sending a package overnight is \$10.75 for the first pound and \$3.95 for each additional pound or portion of a pound. A plastic mailing bag can hold up to 3 pounds. The cost $f(x)$ of sending a package in a plastic mailing bag is given by

$$f(x) = \begin{cases} 10.75, & 0 < x \le 1 \\ 14.70, & 1 < x \le 2 \\ 18.65, & 2 < x \le 3 \end{cases}$$

where x represents the weight of the package (in pounds). Show that the limit of f as $x \to 1$ does not exist.

Limits for Practice

71. ***Communications*** The cost of a telephone call between two cities is \$0.75 for the first minute and \$0.50 for each additional minute or portion of a minute. A model for the cost C is given by $C(t) = 0.75 - 0.50[\![-(t-1)]\!]$ where t is the time in minutes. (Recall from Section 1.6 that $f(x) = [\![x]\!] =$ the greatest integer less than or equal to x.)

(a) Sketch the graph of C for $0 < t \le 5$.

(b) Complete the table and observe the behavior of C as t approaches 3.5. Use the graph from part (a) and the table to find $\lim_{t \to 3.5} C(t)$.

t	3	3.3	3.4	3.5	3.6	3.7	4
C				?			

(c) Complete the table and observe the behavior of C as t approaches 3. Does the limit of $C(t)$ as t approaches 3 exist? Explain.

t	2	2.5	2.9	3	3.1	3.5	4
C				?			

Model It

72. ***Consumer Awareness*** The cost C (in dollars) of making x photocopies at a copy shop is given by the function

$$C(x) = \begin{cases} 0.15x, & 0 < x \le 25 \\ 0.10x, & 25 < x \le 100 \\ 0.07x, & 100 < x \le 500 \\ 0.05x, & x > 500 \end{cases}$$

(a) Sketch a graph of the function.

(b) Find each limit and interpret your result in the context of the situation.

(i) $\lim_{x \to 15} C(x)$ (ii) $\lim_{x \to 99} C(x)$ (iii) $\lim_{x \to 305} C(x)$

(c) Create a table of values to show numerically that each limit does not exist.

(i) $\lim_{x \to 25} C(x)$ (ii) $\lim_{x \to 100} C(x)$ (iii) $\lim_{x \to 500} C(x)$

(d) Explain how you can use the graph in part (a) to verify that the limits in part (c) do not exist.

Synthesis

True or False? **In Exercises 73 and 74, determine whether the statement is true or false. Justify your answer.**

73. When your attempt to find the limit of a rational function yields the indeterminate form $\frac{0}{0}$, the rational function's numerator and denominator have a common factor.

74. If $f(c) = L$, then $\lim_{x \to c} f(x) = L$.

75. ***Think About It***

(a) Sketch the graph of a function for which $f(2)$ is defined but for which the limit of $f(x)$ as x approaches 2 does not exist.

(b) Sketch the graph of a function for which the limit of $f(x)$ as x approaches 1 is 4 but for which $f(1) \ne 4$.

76. ***Writing*** Consider the limit of the rational function given by $p(x)/q(x)$. What conclusion can you make if direct substitution produces each expression? Write a short paragraph explaining your reasoning.

(a) $\lim_{x \to c} \frac{p(x)}{q(x)} = \frac{0}{1}$ (b) $\lim_{x \to c} \frac{p(x)}{q(x)} = \frac{1}{1}$

(c) $\lim_{x \to c} \frac{p(x)}{q(x)} = \frac{1}{0}$ (d) $\lim_{x \to c} \frac{p(x)}{q(x)} = \frac{0}{0}$

Skills Review

77. Write an equation of the line that passes through $(6, -10)$ and is perpendicular to the line that passes through $(4, -6)$ and $(3, -4)$.

78. Write an equation of the line that passes through $(1, -1)$ and is parallel to the line that passes through $(3, -3)$ and $(5, -2)$.

79. ***Arc Length*** Find the length of the arc on a circle with a radius of 8.5 inches intercepted by a central angle of $45°$.

80. ***Arc Length*** Find the length of the arc on a circle with a radius of 26 millimeters intercepted by a central angle of $101°$.

81. ***Circular Sector*** Find the area of the sector of a circle with a radius of 9 centimeters and central angle $\theta = 135°$.

82. ***Circular Sector*** Find the area of the sector of a circle with a radius of 3.4 feet and central angle $\theta = 81°$.

In Exercises 83–88, identify the type of conic algebraically. Then use a graphing utility to graph the conic.

83. $r = \frac{3}{1 + \cos\theta}$

84. $r = \frac{12}{3 + 2\sin\theta}$

85. $r = \frac{9}{2 + 3\cos\theta}$

86. $r = \frac{4}{4 + \cos\theta}$

87. $r = \frac{5}{1 - \sin\theta}$

88. $r = \frac{6}{3 - 4\sin\theta}$

In Exercises 89–92, determine whether the vectors are orthogonal, parallel, or neither.

89. $\langle 7, -2, 3\rangle, \langle -1, 4, 5\rangle$

90. $\langle 5, 5, 0\rangle, \langle 0, 5, 1\rangle$

91. $\langle -4, 3, -6\rangle, \langle 12, -9, 18\rangle$

92. $\langle 2, -3, 1\rangle, \langle -2, 2, 2\rangle$