The Chain Rule … Fact

Example Find the derivative $\frac{d}{dx}\left[(2x+5)^3\right]$.

The Chain Rule … Fact

Example Find the derivative $\frac{d}{dx}\left[(2x+5)^3\right]$.

Solution We begin by viewing $(2x+5)^3$ as a composition of functions and identifying the outside function f and the inside function g. The outside function is the last thing you do when computing the expression for a given input x. Here, the outside function is the cubing function:

$$(2x+5)^3 = f(g(x)), \quad \text{where} \quad f(x) = x^3 \quad \text{and} \quad g(x) = 2x+5.$$

Next, we do the computations required for the chain rule formula:

$$\begin{aligned} f(x) &= x^3 & \qquad g(x) &= 2x+5 \\ f'(x) &= 3x^2 & \qquad g'(x) &= 2 \\ f'(g(x)) &= 3(2x+5)^2 & & \end{aligned}$$

Finally, we use the formula:

$$\begin{array}{ccccc} \frac{d}{dx} & [f(g(x))] & = & f'(g(x)) & \cdot & g'(x) \\ & \downarrow & & \downarrow & & \downarrow \\ \frac{d}{dx} & \left[(2x+5)^3\right] & = & 3(2x+5)^2 & \cdot & 2 \end{array}$$

Chain rule

Statement

The power rule says that

$$\frac{d}{dx}[x^n] = nx^{n-1}.$$

This rule is valid for any power n, but not for any base other than the simple input variable x. For instance,

$$\frac{d}{dx}[(2x+5)^3] \neq 3(2x+5)^2.$$

The function $h(x) = (2x+5)^3$ is built up of the two simpler functions $g(x) = 2x+5$ and $f(x) = x^3$:

$$h(x) = (2x+5)^3 = (g(x))^3 = f(g(x)).$$

Technically speaking, h is the composition of f and g. The next rule expresses the derivative of such a function in terms of the derivatives of its components.

Chain rule. For functions f and g

$$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x).$$

In the composition $f(g(x))$, we call f the outside function and g the inside function. With this terminology, the rule says that the derivative of the composition of two functions is the derivative of the outside function evaluated at the inside function times the derivative of the inside function.