

Differentiation (... Practice Set 2)

Derivatives - Sum, Power, Product, Quotient, Chain Rules

Differentiate each function with respect to x . Problems may contain constants a , b , and c .

1) $f(x) = 3x^5$

2) $f(x) = x$

3) $f(x) = x^3\sqrt{3}$

4) $f(x) = -2x^4$

5) $f(x) = -\frac{1}{4}x^2$

6) $f(x) = -\frac{1}{2}x^5$

7) $f(x) = -5x^{-3}$

8) $f(x) = 5x^{\frac{5}{3}}$

9) $f(x) = -4x^b$

10) $f(x) = -x^{2a}$

11) $y = -x - 3$

12) $y = 5x^2 + 1$

13) $y = -5x^4 - 2x + 4x^{-1}$

14) $y = 2x^4 - 5x^{-1} + 5x^{-3}$

15) $y = -2x^{-2} - 4x^{-4} + 3x^{-5}$

16) $y = -2x^{-1} - \frac{1}{x^3} + 3x^{-4}$

17) $y = 4x^{-1} + 3x^{-2} + \frac{4}{x^4}$

18) $y = -2x^2 + 4x^{-4} + 5x^{-5}$

19) $y = -4x^4 - \frac{2}{x^3} - \frac{5}{x^5}$

20) $y = 3x^3 - \frac{1}{x^3} + 4x^{-4}$

Differentiation (... Practice Set 2)

Answers

Differentiate each function with respect to x . Problems may contain constants a , b , and c .

1) $f(x) = 3x^5$

$$f'(x) = 15x^4$$

2) $f(x) = x$

$$f'(x) = 1$$

3) $f(x) = x^3\sqrt{3}$

$$f'(x) = 3x^2\sqrt{3}$$

4) $f(x) = -2x^4$

$$f'(x) = -8x^3$$

5) $f(x) = -\frac{1}{4}x^2$

$$f'(x) = -\frac{x}{2}$$

6) $f(x) = -\frac{1}{2}x^5$

$$f'(x) = -\frac{5x^4}{2}$$

7) $f(x) = -5x^{-3}$

$$f'(x) = \frac{15}{x^4}$$

8) $f(x) = 5x^{\frac{5}{3}}$ $f'(x) = \frac{25x^{\frac{2}{3}}}{3}$

9) $f(x) = -4x^b$

$$f'(x) = -4bx^{b-1}$$

10) $f(x) = -x^{2a}$

$$f'(x) = -2ax^{2a-1}$$

11) $y = -x - 3$

$$\frac{dy}{dx} = -1$$

12) $y = 5x^2 + 1$

$$\frac{dy}{dx} = 10x$$

13) $y = -5x^4 - 2x + 4x^{-1}$

$$\frac{dy}{dx} = -20x^3 - 2 - \frac{4}{x^2}$$

14) $y = 2x^4 - 5x^{-1} + 5x^{-3}$

$$\frac{dy}{dx} = 8x^3 + \frac{5}{x^2} - \frac{15}{x^4}$$

15) $y = -2x^{-2} - 4x^{-4} + 3x^{-5}$

$$\frac{dy}{dx} = \frac{4}{x^3} + \frac{16}{x^5} - \frac{15}{x^6}$$

16) $y = -2x^{-1} - \frac{1}{x^3} + 3x^{-4}$

$$\frac{dy}{dx} = \frac{2}{x^2} + \frac{3}{x^4} - \frac{12}{x^5}$$

17) $y = 4x^{-1} + 3x^{-2} + \frac{4}{x^4}$

$$\frac{dy}{dx} = -\frac{4}{x^2} - \frac{6}{x^3} - \frac{16}{x^5}$$

18) $y = -2x^2 + 4x^{-4} + 5x^{-5}$

$$\frac{dy}{dx} = -4x - \frac{16}{x^5} - \frac{25}{x^6}$$

19) $y = -4x^4 - \frac{2}{x^3} - \frac{5}{x^5}$

$$\frac{dy}{dx} = -16x^3 + \frac{6}{x^4} + \frac{25}{x^6}$$

20) $y = 3x^3 - \frac{1}{x^3} + 4x^{-4}$

$$\frac{dy}{dx} = 9x^2 + \frac{3}{x^4} - \frac{16}{x^5}$$

Differentiation (... Practice Set 2)

Product Rule:

$$21) y = x^3(3x^5 - 2)$$

$$22) y = x^2(2x^4 - 5)$$

$$23) y = 2x^3(-x^3 + 3)$$

$$24) y = (-4x^4 + 1) \cdot -4x^4$$

$$25) y = (-5x^4 - 3x^3 - 2)(3x^3 - 2)$$

$$26) y = (4x^4 - 5)(-x^4 + x^2 + 2)$$

$$27) y = (-x^5 + 2x^4 + 4)(x^3 + 3)$$

$$28) y = (1 + 3x^{-3})(x^3 + 1)$$

$$29) y = \left(1 + \frac{1}{x^2}\right)(5x^2 + 3)$$

$$30) y = (-5 - x^{-3})(x^5 + 2)$$

$$31) y = \left(1 + \frac{1}{x^5}\right)(3x^4 - 2)$$

Differentiation (... Practice Set 2)

Answers

Product Rule:

$$21) y = x^3(3x^5 - 2)$$

$$\begin{aligned}\frac{dy}{dx} &= x^3 \cdot 15x^4 + (3x^5 - 2) \cdot 3x^2 \\ &= 24x^7 - 6x^2\end{aligned}$$

$$23) y = 2x^3(-x^3 + 3)$$

$$\begin{aligned}\frac{dy}{dx} &= 2x^3 \cdot -3x^2 + (-x^3 + 3) \cdot 6x^2 \\ &= -12x^5 + 18x^2\end{aligned}$$

$$25) y = (-5x^4 - 3x^3 - 2)(3x^3 - 2)$$

$$\begin{aligned}\frac{dy}{dx} &= (-5x^4 - 3x^3 - 2) \cdot 9x^2 + (3x^3 - 2)(-20x^3 - 9x^2) \\ &= -105x^6 - 54x^5 + 40x^3\end{aligned}$$

$$26) y = (4x^4 - 5)(-x^4 + x^2 + 2)$$

$$\begin{aligned}\frac{dy}{dx} &= (4x^4 - 5)(-4x^3 + 2x) + (-x^4 + x^2 + 2) \cdot 16x^3 \\ &= -32x^7 + 24x^5 + 52x^3 - 10x\end{aligned}$$

$$27) y = (-x^5 + 2x^4 + 4)(x^3 + 3)$$

$$\begin{aligned}\frac{dy}{dx} &= (-x^5 + 2x^4 + 4) \cdot 3x^2 + (x^3 + 3)(-5x^4 + 8x^3) \\ &= -8x^7 + 14x^6 - 15x^4 + 24x^3 + 12x^2\end{aligned}$$

$$28) y = (1 + 3x^{-3})(x^3 + 1)$$

$$\begin{aligned}\frac{dy}{dx} &= (1 + 3x^{-3}) \cdot 3x^2 + (x^3 + 1) \cdot -9x^{-4} \\ &= 3x^2 - \frac{9}{x^4}\end{aligned}$$

$$30) y = (-5 - x^{-3})(x^5 + 2)$$

$$\begin{aligned}\frac{dy}{dx} &= (-5 - x^{-3}) \cdot 5x^4 + (x^5 + 2) \cdot 3x^{-4} \\ &= -25x^4 - 2x + \frac{6}{x^4}\end{aligned}$$

$$22) y = x^2(2x^4 - 5) \quad \frac{dy}{dx} = x^2 \cdot 8x^3 + (2x^4 - 5) \cdot 2x$$
$$= 12x^5 - 10x$$

$$24) y = (-4x^4 + 1) \cdot -4x^4$$

$$\begin{aligned}\frac{dy}{dx} &= (-4x^4 + 1) \cdot -16x^3 - 4x^4 \cdot -16x^3 \\ &= 128x^7 - 16x^3\end{aligned}$$

$$29) y = \left(1 + \frac{1}{x^2}\right)(5x^2 + 3)$$

$$\begin{aligned}\frac{dy}{dx} &= (1 + x^{-2}) \cdot 10x + (5x^2 + 3) \cdot -2x^{-3} \\ &= 10x - \frac{6}{x^3}\end{aligned}$$

$$31) y = \left(1 + \frac{1}{x^5}\right)(3x^4 - 2)$$

$$\begin{aligned}\frac{dy}{dx} &= (1 + x^{-5}) \cdot 12x^3 + (3x^4 - 2) \cdot -5x^{-6} \\ &= 12x^3 - \frac{3}{x^2} + \frac{10}{x^6}\end{aligned}$$

Differentiation (... Practice Set 2)

Quotient Rule:

$$32) y = \frac{x^4}{4x^2 + 4}$$

$$33) y = \frac{x^3}{5x^2 - 4}$$

$$34) y = \frac{5x^4 + 1}{4x^5 + 3}$$

$$35) y = \frac{3x^3 - 3x^2}{3x^3 + 5}$$

$$36) y = \frac{x^4 + 4x^2 - 4}{2x^3 - 4}$$

$$37) y = \frac{4x^5 + 5x^3 - 2x^2}{4x^4 + 3}$$

$$38) y = \frac{3x^5 - 5x^4 - x^2}{4x^5 - 4}$$

$$39) y = \frac{2x^5 + 2x^3 - 5x^2}{4x^4 - 4}$$

$$40) y = \frac{5x^5 - x^3 - 4}{2x^2 - 5}$$

Differentiation (... Practice Set 2)

Answers

Quotient Rule:

$$32) y = \frac{x^4}{4x^2 + 4} \quad \frac{dy}{dx} = \frac{(4x^2 + 4) \cdot 4x^3 - x^4 \cdot 8x}{(4x^2 + 4)^2}$$

$$= \frac{x^5 + 2x^3}{2x^4 + 4x^2 + 2}$$

$$33) y = \frac{x^3}{5x^2 - 4} \quad \frac{dy}{dx} = \frac{(5x^2 - 4) \cdot 3x^2 - x^3 \cdot 10x}{(5x^2 - 4)^2}$$

$$= \frac{5x^4 - 12x^2}{25x^4 - 40x^2 + 16}$$

$$34) y = \frac{5x^4 + 1}{4x^5 + 3}$$

$$\frac{dy}{dx} = \frac{(4x^5 + 3) \cdot 20x^3 - (5x^4 + 1) \cdot 20x^4}{(4x^5 + 3)^2}$$

$$= \frac{-20x^8 - 20x^4 + 60x^3}{16x^{10} + 24x^5 + 9}$$

$$35) y = \frac{3x^3 - 3x^2}{3x^3 + 5}$$

$$\frac{dy}{dx} = \frac{(3x^3 + 5)(9x^2 - 6x) - (3x^3 - 3x^2) \cdot 9x^2}{(3x^3 + 5)^2}$$

$$= \frac{9x^4 + 45x^2 - 30x}{9x^6 + 30x^3 + 25}$$

$$36) y = \frac{x^4 + 4x^2 - 4}{2x^3 - 4}$$

$$\frac{dy}{dx} = \frac{(2x^3 - 4)(4x^3 + 8x) - (x^4 + 4x^2 - 4) \cdot 6x^2}{(2x^3 - 4)^2}$$

$$= \frac{x^6 - 4x^4 - 8x^3 + 12x^2 - 16x}{2x^6 - 8x^3 + 8}$$

$$37) y = \frac{4x^5 + 5x^3 - 2x^2}{4x^4 + 3}$$

$$\frac{dy}{dx} = \frac{(4x^4 + 3)(20x^4 + 15x^2 - 4x) - (4x^5 + 5x^3 - 2x^2) \cdot 16x^3}{(4x^4 + 3)^2}$$

$$= \frac{16x^8 - 20x^6 + 16x^5 + 60x^4 + 45x^2 - 12x}{16x^8 + 24x^4 + 9}$$

$$38) y = \frac{3x^5 - 5x^4 - x^2}{4x^5 - 4}$$

$$\frac{dy}{dx} = \frac{(4x^5 - 4)(15x^4 - 20x^3 - 2x) - (3x^5 - 5x^4 - x^2) \cdot 20x^4}{(4x^5 - 4)^2}$$

$$= \frac{5x^8 + 3x^6 - 15x^4 + 20x^3 + 2x}{4x^{10} - 8x^5 + 4}$$

$$39) y = \frac{2x^5 + 2x^3 - 5x^2}{4x^4 - 4}$$

$$\frac{dy}{dx} = \frac{(4x^4 - 4)(10x^4 + 6x^2 - 10x) - (2x^5 + 2x^3 - 5x^2) \cdot 16x^3}{(4x^4 - 4)^2}$$

$$= \frac{x^8 - x^6 + 5x^5 - 5x^4 - 3x^2 + 5x}{2x^8 - 4x^4 + 2}$$

$$40) y = \frac{5x^5 - x^3 - 4}{2x^2 - 5}$$

$$\frac{dy}{dx} = \frac{(2x^2 - 5)(25x^4 - 3x^2) - (5x^5 - x^3 - 4) \cdot 4x}{(2x^2 - 5)^2}$$

$$= \frac{30x^6 - 127x^4 + 15x^2 + 16x}{4x^4 - 20x^2 + 25}$$

Differentiation (... Practice Set 2)

Chain Rule:

$$41) y = (x^4 - 5)^4$$

$$42) y = (-3x^3 - 5)^2$$

$$43) y = (2x - 1)^3$$

$$44) y = (-3x^5 - 1)^5$$

$$45) y = (-4x^2 - 5)^4$$

$$46) y = (-x + 4)^2$$

$$47) y = ((3x^4 + 5)^2 + 5)^4$$

$$48) y = ((-x^5 + 2)^5 - 5)^2$$

$$49) y = ((5x^4 + 3)^4 + 1)^2$$

$$50) y = ((5x^2 - 3)^5 + 2)^4$$

Differentiation (... Practice Set 2)

Answers

Chain Rule:

$$41) y = (x^4 - 5)^4$$

$$\begin{aligned}\frac{dy}{dx} &= 4(x^4 - 5)^3 \cdot 4x^3 \\ &= 16x^3(x^4 - 5)^3\end{aligned}$$

$$42) y = (-3x^3 - 5)^2$$

$$\begin{aligned}\frac{dy}{dx} &= 2(-3x^3 - 5) \cdot -9x^2 \\ &= -18x^2(-3x^3 - 5)\end{aligned}$$

$$43) y = (2x - 1)^3$$

$$\begin{aligned}\frac{dy}{dx} &= 3(2x - 1)^2 \cdot 2 \\ &= 6(2x - 1)^2\end{aligned}$$

$$44) y = (-3x^5 - 1)^5 \quad \frac{dy}{dx} = 5(-3x^5 - 1)^4 \cdot -15x^4$$
$$= -75x^4(-3x^5 - 1)^4$$

$$45) y = (-4x^2 - 5)^4$$

$$\begin{aligned}\frac{dy}{dx} &= 4(-4x^2 - 5)^3 \cdot -8x \\ &= -32x(-4x^2 - 5)^3\end{aligned}$$

$$46) y = (-x + 4)^2$$

$$\begin{aligned}\frac{dy}{dx} &= 2(-x + 4) \cdot -1 \\ &= -2(-x + 4)\end{aligned}$$

$$47) y = ((3x^4 + 5)^2 + 5)^4$$

$$\begin{aligned}\frac{dy}{dx} &= 4((3x^4 + 5)^2 + 5)^3 \cdot 2(3x^4 + 5) \cdot 12x^3 \\ &= 96x^3((3x^4 + 5)^2 + 5)^3(3x^4 + 5)\end{aligned}$$

$$48) y = ((-x^5 + 2)^5 - 5)^2$$

$$\begin{aligned}\frac{dy}{dx} &= 2((-x^5 + 2)^5 - 5) \cdot 5(-x^5 + 2)^4 \cdot -5x^4 \\ &= -50x^4(-x^5 + 2)^4((-x^5 + 2)^5 - 5)\end{aligned}$$

$$49) y = ((5x^4 + 3)^4 + 1)^2$$

$$\begin{aligned}\frac{dy}{dx} &= 2((5x^4 + 3)^4 + 1) \cdot 4(5x^4 + 3)^3 \cdot 20x^3 \\ &= 160x^3(5x^4 + 3)^3((5x^4 + 3)^4 + 1)\end{aligned}$$

$$50) y = ((5x^2 - 3)^5 + 2)^4$$

$$\begin{aligned}\frac{dy}{dx} &= 4((5x^2 - 3)^5 + 2)^3 \cdot 5(5x^2 - 3)^4 \cdot 10x \\ &= 200x((5x^2 - 3)^5 + 2)^3 \cdot (5x^2 - 3)^4\end{aligned}$$