- The **product rule** is used to differentiate a function that is the multiplication of two functions.

$$f(x) = u(x) \times v(x)$$

⟹

$$\boxed{f'(x) = u'v + v'u}$$

$$\boxed{\frac{dy}{dx} = \left(\frac{du}{dx} \times v\right) + \left(\frac{dv}{dx} \times u\right)}$$

Example

$$f(x) = (3x-5) \times (4x+7)$$

$$u = 3x-5 \qquad v = 4x+7$$
$$u' = 3 \qquad\quad v' = 4$$

$$f'(x) = 3(4x+7) + 4(3x-5)$$
$$f'(x) = 12x+21+12x-20 = 24x+1$$
$$\color{red}{f'(x) = 24x+1}$$

- The **quotient rule** is used to differentiate a function that is the division of two functions.

$$f(x) = \frac{u(x)}{v(x)}$$

⇨

$$f'(x) = \frac{u'v - v'u}{v^2}$$

$$\frac{dy}{dx} = \left(\frac{du}{dx} \times v\right) + \left(\frac{dv}{dx} \times u\right)$$

Example

$$f(x) = \frac{3x-5}{4x+7}$$

$$u = 3x-5 \qquad v = 4x+7$$
$$u' = 3 \qquad\quad v' = 4$$

$$f'(x) = \frac{3(4x+7) - 4(3x-5)}{(4x+7)^2} = \frac{12x+21-12x+20}{(4x+7)^2}$$

$$f'(x) = \frac{41}{(4x+7)^2}$$

- Find the derivative: $f(x)=(x^2+x)(3x-1)$

$$\boxed{f'(x)=u'v+v'u}$$

Example

$u=x^2+x, \quad u'=2x+1$

$v=3x+1, \quad v'=3$

$f'(x)=(2x+1)(3x-1)+3(x^2+x)$

$f'(x)=(6x^2+x+1)+(3x^2+3x)$

$f'(x)=9x^2+4x+1$

• Find the derivative: $y=\sqrt{5x^2-6}$

$y=\sqrt{u}=u^{\frac{1}{2}} \qquad u=5x^2-6$

$\dfrac{dy}{du}=\dfrac{1}{2}u^{-\frac{1}{2}} \qquad \dfrac{du}{dx}=10x$

$\dfrac{dy}{dx}=\dfrac{dy}{du}\times\dfrac{du}{dx}=\dfrac{1}{2}u^{-\frac{1}{2}}\times10x$

$\dfrac{dy}{dx}=\dfrac{1}{2}(5x^2-6)^{-\frac{1}{2}}\times10x$

$\dfrac{dy}{dx}=\dfrac{10x}{2\sqrt{5x^2-6}}=\dfrac{5x}{\sqrt{5x^2-6}}$

Recall:

$$\boxed{\dfrac{dy}{dx}=\dfrac{dy}{du}\cdot\dfrac{du}{dx}}$$

- Find the derivative: $y = \dfrac{2x}{(x-6)^2}$

$$f'(x) = \dfrac{u'v - v'u}{v^2}$$

$u = 2x, \quad v = (x-6)^2$

$u' = 2, \quad v' = 2(x-6)$

$$f'(x) = \dfrac{u'v - v'u}{v^2} = \dfrac{2(x-6)^2 - 2(x-6)(2x)}{((x-6)^2)^2}$$

$$f'(x) = \dfrac{2(x-6)^2 - 4x(x-6)}{(x-6)^4} = \dfrac{2(x-6) - 4x}{(x-6)^3} = \dfrac{2x - 12 - 4x}{(x-6)^3}$$

$$f'(x) = \dfrac{-2(x+6)}{(x-6)^3}$$