Derivatives of Exponential Functions

Differentiate each function with respect to x.

1)
$$f(x) = e^{5x^4}$$
 2) $f(x) = e^{3x^5}$

3)
$$f(x) = e^{5x^2}$$
 4) $f(x) = e^{4x^3}$

5)
$$f(x) = e^{3x^2}$$
 6) $f(x) = e^{4x^2}$

Answers

Derivatives of Exponential Functions

- 1) $f'(x) = e^{5x^4} \cdot 20x^3$ 2) $f'(x) = e^{3x^5} \cdot 15x^4$
- 3) $f'(x) = e^{5x^2} \cdot 10x$ 4) $f'(x) = e^{4x^3} \cdot 12x^2$
- 5) $f'(x) = e^{3x^2} \cdot 6x$ 6) $f'(x) = e^{4x^2} \cdot 8x$

7)
$$f(x) = e^{e^{2x^2}}$$

8) $f(x) = \frac{e^{5x^2}}{e^{2x^4 - 1}}$

9)
$$f(x) = e^{3x^2}(3x^5 - 4)$$
 10) $f(x) = (3x^3 + 5) \cdot e^{2x^2}$

11)
$$f(x) = \frac{e^{3x^4}}{2x^2 + 1}$$
 12) $f(x) = \frac{x^4 + 3}{e^{5x^5}}$

13)
$$f(x) = 2^{3x^3}$$
 14) $f(x) = 5^{x^4}$

15)
$$f(x) = 3^{5x^2}$$
 16) $f(x) = 4^{2x^4}$

Answers

Derivatives of Exponential Functions 7) $f'(x) = e^{e^{2x^3}} \cdot e^{2x^3} \cdot 6x^2$ $= 6x^2 e^{e^{2x^3} + 2x^3}$ $= 0x^2 e^{e^{2x^3} + 2x^3}$ $= 2x e^{5x^2 - (2x^4 - 1)}(10x - 8x^3)$ $= 2x e^{5x^2 - 2x^4 + 1}(5 - 4x^2)$ 9) $f'(x) = e^{3x^2} \cdot 15x^4 + (3x^5 - 4) \cdot e^{3x^3} \cdot 6x$ $= 3x e^{3x^2}(5x^3 + 6x^5 - 8)$ 10) $f'(x) = (3x^3 + 5) \cdot e^{2x^2} \cdot 4x + e^{2x^2} \cdot 9x^2$ $= x e^{2x^2}(12x^3 + 20 + 9x)$ 11) $f'(x) = \frac{(2x^2 + 1) \cdot e^{3x^4} \cdot 12x^3 - e^{3x^4} \cdot 4x}{(2x^2 + 1)^2}$ $= \frac{4x e^{3x^4}(6x^4 + 3x^2 - 1)}{(2x^2 + 1)^2}$ 12) $f'(x) = \frac{e^{5x^5} \cdot 4x^3 - (x^4 + 3) \cdot e^{5x^5} \cdot 25x^4}{(e^{5x^5})^2}$ $= \frac{x^3(4 - 25x^5 - 75x)}{e^{5x^5}}$ 13) $f'(x) = 2^{3x^3} \ln 2 \cdot 9x^2$ 14) $f'(x) = 5^{x^4} \ln 5 \cdot 4x^3$ 15) $f'(x) = 3^{5x^2} \ln 3 \cdot 10x$ 16) $f'(x) = 4^{2x^4} \ln 4 \cdot 8x^3$ $= 2x^3 \cdot 4^{2x^4 + 1} \ln 4$

Solve each equation.

17)
$$3^{3b} = 243$$
 18) $16^m = 64$

19)
$$\log_2(2p+1) = \log_2(5p-2)$$
 20) $\log 3 + \log (x-7) = 1$

Condense each expression to a single logarithm.

21)
$$20 \log_2 u - 4 \log_2 v$$

22) $\frac{\log_5 u}{2} + \frac{\log_5 v}{2} + \frac{\log_5 w}{2}$

Expand each logarithm.

23) $\log_9 \left(a \cdot b \cdot c^3 \right)$ 24) $\log_8 \left(\frac{x}{v^6} \right)^6$

Solve each related rate problem.

25) A 17 ft ladder is leaning against a wall and sliding towards the floor. The foot of the ladder is sliding away from the base of the wall at a rate of 7 ft/sec. How fast is the top of the ladder sliding down the wall when the top of the ladder is 8 ft from the ground?

Answers

Derivatives of Exponential Functions 17) $\left(\frac{5}{3}\right)$ 18) $\left(\frac{3}{2}\right)$ 19) $\left\{1\right\}$ 20) $\left(\frac{31}{3}\right)$ 21) $\log_2 \frac{u^{20}}{v^4}$ 22) $\log_5 \sqrt{wvu}$ 23) $\log_9 a + \log_9 b + 3\log_9 c$ 24) $6\log_8 x - 36\log_8 y$ 25) $\frac{105}{8}$ ft/sec

Solve each optimization problem. You may use the provided box to sketch the problem setup if necessary.

26) A graphic designer is asked to create a movie poster with a 98 in² photo surrounded by a 4 in border at the top and bottom and a 2 in border on each side. What overall dimensions for the poster should the designer choose to use the least amount of paper?

For each problem, find all points of relative minima and maxima.

27)
$$f(x) = \frac{x^2}{2} + 1$$
 28) $f(x) = x^3 - 3x^2$

For each problem, find the open intervals where the function is concave up and concave down. 29) $y = x^3 - 4x^2 + 3$

30)
$$y = \frac{3}{x+1}$$

Answers

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26) 11 in wide by 22 in tall

- 27) Relative minimum: (0, 1)
 No relative maxima.
 28) Relative minimum: (2, -4)
 Relative maximum: (0, 0)
- 29) Concave up: $\left(\frac{4}{3}, \infty\right)$ Concave down: $\left(-\infty, \frac{4}{3}\right)$
- 30) Concave up: $(-1, \infty)$ Concave down: $(-\infty, -1)$