The 1st Derivative Test

… Set 4

1st & 2nd Derivative Test Review Sheet   Calculus Honors

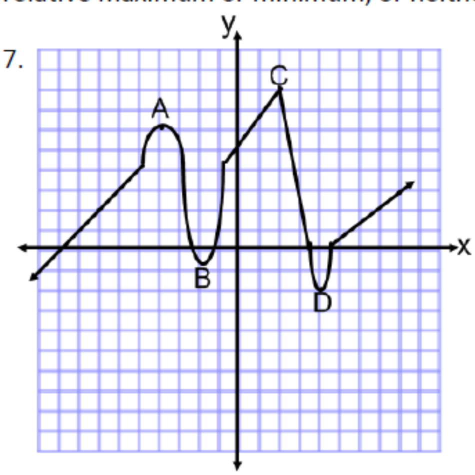
Review Sheet: Applications of Derivatives

In the examples below, locate the absolute extrema of a function on the indicated closed interval.

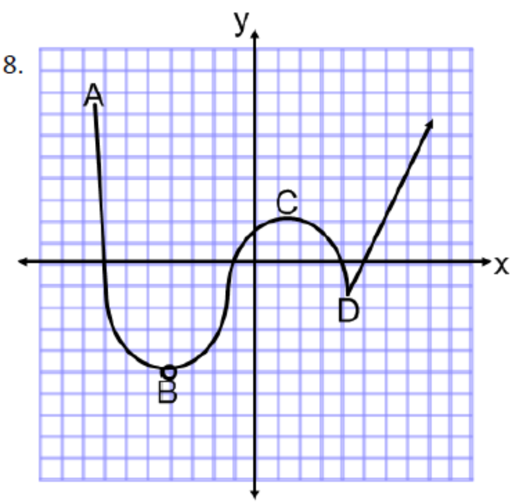
1. $f(x) = \frac{2x+5}{3}, [0,5]$
2. $f(x) = -x^2 + 3x, [0,3]$
3. $y = 3x^{2/3} - \frac{3}{2}x^2, [-1,2]$
4. $h(s) = \frac{1}{s-2}, [0,1]$
5. $f(x) = 2\sin x - \cos 2x, [0,2\pi]$
6. $g(x) = \sec x, \left[-\frac{\pi}{6}, \frac{\pi}{3}\right]$

In the examples below, decide whether each labeled point is an absolute maximum or minimum, a relative maximum or minimum, or neither.

7.



8.



In the examples below, determine whether Rolle's Theorem can be applied to f on the closed interval [a,b]. If Rolle's Theorem can be applied, find all values c in the open interval (a, b) such that f'(c) = 0.

9. $f(x) = x^2 - 2x, [0,2]$
10. $f(x) = \sin x, [0,2\pi]$
11. $f(x) = \frac{x^2-1}{x}, [-1,1]$

In the examples below, use the Mean Value Theorem to solve the word problems.

12. When an object is removed from a furnace and placed in an environment with a constant temperature of 90°F, its core temperature is 1500°F. Five hours later the core temperature is 390°F. Explain why there must exist a time in the interval when the temperature is decreasing at a rate of 222°F per hour.

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Answers

Answer Key:

1.) (0, 5/3) minimum; (5,5) maximum

2.) (0,0)& (3, 0) minimum; (3/2, 9/4) maximum

3.) (2, -1.2378) minimum; (-1, 1.5) maximum

4.)(0, -1/2) max; (1,-1) min

5.) (0,-1), $\left(\frac{3\pi}{2}, -1\right)$, $(2\pi, -1)$ min; $\left(\frac{\pi}{2}, 3\right)$ max

6.) (0,1) min; $\left(\frac{\pi}{3}, 2\right)$ max

7.) A: relative max; B: relative min; C: absolute max; D: absolute min

8.) A: absolute max; B: neither; C: relative max; D: relative min

9.) Rolle's Theorem can be applied; c = 1

10.) Rolle's Theorem can be applied; $c = \left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$

11.) Rolle's Theorem cannot be applied because it's not a continuous function (discontinuous @ x = 0)

12.) When the Mean Value Theorem is applied, the answer is -222°F/hour. Since that is the average throughout the interval, there has to be a least one time when it's decreasing at that rate.

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13. A 9:13 a sports car is traveling 35 miles per hour. Two minutes later, the car is traveling at 85 miles per hour. Prove that at some time during this two-minute interval, the car's acceleration is exactly 1500 miles per hour squared.

In the examples below, (a) find the critical numbers of f (if any), (b) find the open interval(s) on which the function is increasing or decreasing, and (c) apply the First Derivative Test to identify all relative extrema.

14. $f(x) = x^2 - 6x$

15. $f(x) = 2x^3 + 3x^2 - 12x$

16. $f(x) = \frac{x^5 - 5x}{5}$

In the examples below, consider the function on the interval $(0, 2\pi)$, (a) find the critical numbers of f (if any), (b) find the open interval(s) on which the function is increasing or decreasing, and (c) apply the First Derivative Test to identify all relative extrema.

17. $f(x) = \sin x + \cos x$

18. $f(x) = \sqrt{3}\sin x + \cos x$

19. $f(x) = \sin^2 x + \sin x$

In the examples below, find the points of inflection and discuss the concavity of the graph of the function.

20. $f(x) = x^3 - 6x^2 + 12x$

21. $f(x) = 2x^3 - 3x^2 - 12x + 5$

22. $f(x) = \sin\frac{x}{2}, [0, 4\pi]$

In the examples below, find all relative extrema. Use the Second Derivative Test.

23. $f(x) = x^4 - 4x^3 + 2$

24. $f(x) = \sqrt{x^2 + 1}$

25. $f(x) = \cos x - x, [0, 4\pi]$

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Answers

13.) When the Mean Value Theorem is applied, the answer is 1500 miles per hour^2. Since that is the average throughout the interval, there has to be a least one time when it's increasing at that rate.

14.) decreasing: $(-\infty, 3)$; increasing: (3, ∞); minimum @ (3, -9)

15.) increasing: $(-\infty, -2)$ & $(1, \infty)$; decreasing: (-2, 1); max @ (-2, 20); min @ (1,-7)

16.) increasing: $(-\infty, -1)$ & $(1, \infty)$; decreasing: $(-1,1)$; max @ (-1, 0.8); min @ (1, -0.8)

17.) increasing: $\left(0, \frac{\pi}{4}\right)$ & $\left(\frac{5\pi}{4}, 2\pi\right)$; decreasing: $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$; max @ $\left(\frac{\pi}{4}, \sqrt{2}\right)$, min @ $\left(\frac{5\pi}{4}, -\sqrt{2}\right)$

18.) increasing: $\left(0, \frac{\pi}{3}\right)$ & $\left(\frac{4\pi}{3}, 2\pi\right)$; decreasing: $\left(\frac{\pi}{3}, \frac{4\pi}{3}\right)$; max @$\left(\frac{\pi}{3}, 2\right)$, min @ $\left(\frac{4\pi}{3}, -2\right)$

19.) increasing: $\left(0, \frac{\pi}{2}\right), \left(\frac{7\pi}{6}, \frac{3\pi}{2}\right), \left(\frac{11\pi}{6}, 2\pi\right)$; decreasing: $\left(\frac{\pi}{2}, \frac{7\pi}{6}\right), \left(\frac{3\pi}{2}, \frac{11\pi}{6}\right)$; max: $\left(\frac{\pi}{2}, 2\right), \left(\frac{3\pi}{2}, 0\right)$; min: $\left(\frac{7\pi}{6}, -\frac{1}{4}\right), \left(\frac{11\pi}{6}, -\frac{1}{4}\right)$