Antiderivatives (… Introduction to Integration) … Set 1

Antiderivatives

We begin a new process called antidifferentiation that is considered the inverse application to differentiation.

If given a function, we can find the derivative. For example, find $\frac{d}{dx}\left(x^3\right)$.

Now suppose you were given a function $f(x) = 3x^2$, can you find a function F such that when you take the derivative of F, we get the result $f(x) = 3x^2$?

We say that $F(x) = x^3$ is the antiderivative of $f(x) = 3x^2$. Can you find any other functions such that the derivative is $f(x) = 3x^2$?

We determine that any function of the form $F(x) = x^3 + C$ where C is any constant, is in fact an antiderivative of $f(x) = 3x^2$.

The operation of determining the original function from its derivative is the inverse operation of differentiation. It is called **antidifferentiation**.

Definition of Antiderivative

A function F is an **antiderivative** of a function f if for every x in the domain of f, it follows that $F'(x)=f(x)$.

Notation for Antiderivatives and Indefinite Integrals:

We call the process of antidifferentiation, **integration**. It is denoted by the integral sign $\int$.

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The symbol $\int f(x)dx$ is called the **indefinite integral** of *f(x)* and it denotes the family of antiderivatives of *f(x)*.

Thus when *F'(x)=f(x)* for all *x*, then we can write the following statement.

$$\int f(x)dx = F(x) + C.$$

The function *f(x)* is called the **integrand** and the *dx* is called the **differential**. We call *C* the **constant of integration**.

The differential identifies our variable of integration. We say $\int f(x)dx$ is the antiderivative of *f* *"with respect to the variable x."*

Integral Notation for Antiderivatives

The notation

$$\int f(x)dx = F(x) + C$$

where *C* is an arbitrary constant, means that *F* is an antiderivative of *f*. That is, *F'(x)=f(x)* for all *x* in the domain of *f*.

Examples:

Since $\frac{d}{dx}\left(x^2\right) = 2x$, then it is true that $\int 2x dx = x^2 + C$.

If $\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2}$, then it is true that $\int \frac{-1}{x^2} dx = ?$

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Note the inverse relationship between derivatives and antiderivatives:

$$\frac{d}{dx}\left[\int f(x)dx\right] = f(x)$$

$$\int f'(x)dx = f(x) + C$$

We can use these relationships to get the following basic integration rules

Basic Integration Rules		
1.	$\int kdx = kx + C$, k is a constant	Constant Rule
2.	$\int kf(x)dx = k\int f(x)dx$	Constant Multiple Rule
3.	$\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$	Sum Rule
4.	$\int [f(x) - g(x)]dx = \int f(x)dx - \int g(x)dx$	Difference Rule
5.	$\int x^n dx = \frac{1}{n+1}x^{n+1} + C$, $n \neq -1$	Simple Power Rule

Explain why the Simple Power Rule makes sense.

Also explain why the Simple Power Rule does only works for $n \neq -1$.

Determine what happens when $n=1$.
Find

$$\int x^{-1}dx = \int \frac{1}{x}dx =$$

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Examples:

Evaluate the indefinite integrals.

$\int x^5 dx$
$\int 3x^4 dx$
$\int 5dx$
$\int -2dx$
$\int 7xdx$
$\int 5x^3 + 9xdx$

Discuss what happens to the constant of integration on the last example.

Sometimes it is important to rewrite a function (the integrand) before we differentiate.

$\int \sqrt{x}dx = \int x^{1/2}dx =$

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Examples:

Evaluate.

$\int \frac{1}{x^3}\,dx$
$\int \sqrt[3]{x}\,dx$
$\int \frac{5}{x^9}\,dx$
$\int \frac{3}{x}\,dx$
$\int \frac{5x^3+7x}{x^2}\,dx$ (Be careful about what rules we have to work with!)
$\int x^2\left(4x^3-6x+1\right)dx$

We also have some antiderivatives that come from exponential or trig functions. Evaluate the following.

Find *f(x)* if *f'(x)* is given.

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$f'(x) = e^x$
$f'(x) = \cos x$
$f'(x) = \sin x$
$f'(x) = \sec x \tan x$
$f'(x) = \dfrac{1}{1+x^2}$
$f''(x) = 5x^2 - 3e^x + 2$

Particular Solutions

We know that there are many solutions to $\int f(x)dx$. However, sometimes we are in a situation where we know more information about a particular solution. We have more information about the problem and are looking for a specific function (instead of a family of functions.)

To illustrate this, we will look at an example.
Consider the function $y = F(x) = \int (3x^2 - 1)dx = x^3 - x + C$

We say that each of the antiderivatives (for the various values of C) is a solution to the *differential equation* $dy/dx = 3x^2 - 1$. We say that a **differential equation** in x and y is an equation that involves x, y, and the derivatives of y. The **general solution** of $dy/dx = 3x^2 - 1$ is $F(x) = x^3 - x + C$.

If we are given more information, such as an **initial condition** (information about the function $F(x)$ for one value of x), then we can determine a **particular solution**.