

Differentiation - Inverse Trigonometric Functions

Differentiate each function with respect to x .

1) $y = \cos^{-1} -5x^3$

2) $y = \sin^{-1} -2x^2$

3) $y = \tan^{-1} 2x^4$

4) $y = \csc^{-1} 4x^2$

5) $y = (\sin^{-1} 5x^2)^3$

6) $y = \sin^{-1} (3x^5 + 1)^3$

7) $y = (\cos^{-1} 4x^2)^2$

8) $y = \cos^{-1} (-2x^3 - 3)^3$

Answers

Differentiation - Inverse Trigonometric Functions

Differentiate each function with respect to x .

1) $y = \cos^{-1} -5x^3$

$$\begin{aligned}\frac{dy}{dx} &= -\frac{1}{\sqrt{1 - (-5x^3)^2}} \cdot -15x^2 \\ &= \frac{15x^2}{\sqrt{1 - 25x^6}}\end{aligned}$$

2) $y = \sin^{-1} -2x^2$

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{\sqrt{1 - (-2x^2)^2}} \cdot -4x \\ &= -\frac{4x}{\sqrt{1 - 4x^4}}\end{aligned}$$

3) $y = \tan^{-1} 2x^4$

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{(2x^4)^2 + 1} \cdot 8x^3 \\ &= \frac{8x^3}{4x^8 + 1}\end{aligned}$$

4) $y = \csc^{-1} 4x^2$

$$\begin{aligned}\frac{dy}{dx} &= -\frac{1}{|4x^2| \sqrt{(4x^2)^2 - 1}} \cdot 8x \\ &= -\frac{2}{x\sqrt{16x^4 - 1}}\end{aligned}$$

5) $y = (\sin^{-1} 5x^2)^3$

$$\begin{aligned}\frac{dy}{dx} &= 3 \cdot (\sin^{-1} 5x^2)^2 \cdot \frac{1}{\sqrt{1 - (5x^2)^2}} \cdot 10x \\ &= \frac{30x \cdot (\sin^{-1} 5x^2)^2}{\sqrt{1 - 25x^4}}\end{aligned}$$

6) $y = \sin^{-1} (3x^5 + 1)^3$

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{\sqrt{1 - ((3x^5 + 1)^3)^2}} \cdot 3(3x^5 + 1)^2 \cdot 15x^4 \\ &= \frac{45x^4(3x^5 + 1)^2}{\sqrt{1 - (3x^5 + 1)^6}}\end{aligned}$$

7) $y = (\cos^{-1} 4x^2)^2$

$$\begin{aligned}\frac{dy}{dx} &= 2\cos^{-1} 4x^2 \cdot -\frac{1}{\sqrt{1 - (4x^2)^2}} \cdot 8x \\ &= -\frac{16x\cos^{-1} 4x^2}{\sqrt{1 - 16x^4}}\end{aligned}$$

8) $y = \cos^{-1} (-2x^3 - 3)^3$

$$\begin{aligned}\frac{dy}{dx} &= -\frac{1}{\sqrt{1 - ((-2x^3 - 3)^3)^2}} \cdot 3(-2x^3 - 3)^2 \cdot -6x^2 \\ &= \frac{18x^2(-2x^3 - 3)^2}{\sqrt{1 - (-2x^3 - 3)^6}}\end{aligned}$$