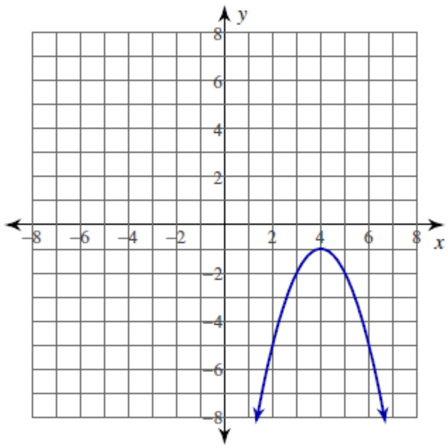
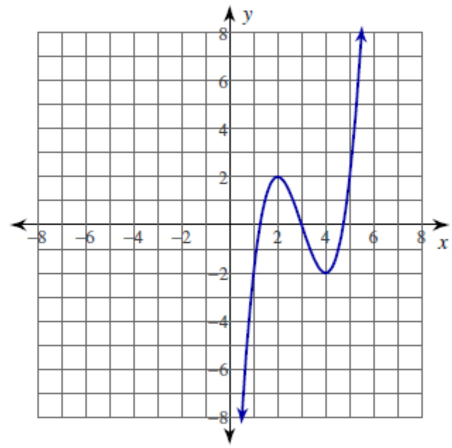
Mean Value Theorem

For each problem, find the values of *c* that satisfy the Mean Value Theorem.

1) $y = -x^2 + 8x - 17;\quad [3, 6]$



2) $y = x^3 - 9x^2 + 24x - 18;\quad [2, 4]$



3) $y = -\dfrac{x^2}{2} + x - \dfrac{1}{2};\quad [-2, 1]$

4) $y = \dfrac{x^2}{2} - 2x - 1;\quad [-1, 1]$

5) $y = x^3 + 3x^2 - 2;\quad [-2, 0]$

6) $y = -x^3 + 4x^2 - 3;\quad [0, 4]$

7) $y = \dfrac{x^2 - 9}{3x};\quad [1, 4]$

8) $y = \dfrac{x^2}{2x - 4};\quad [-4, 1]$

Answers

Mean Value Theorem

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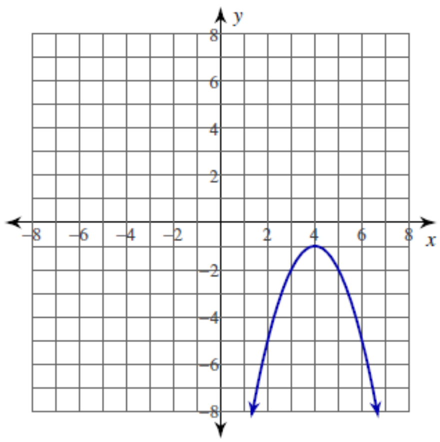
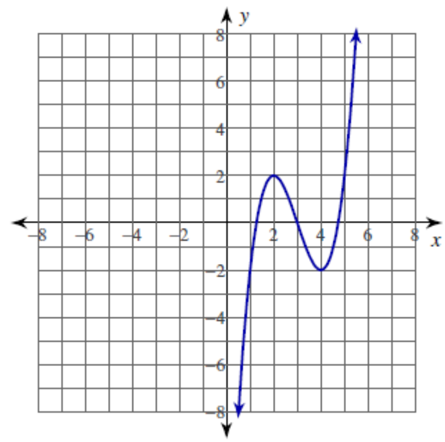


$\left\{\frac{9}{2}\right\}$

2) $y = x^3 - 9x^2 + 24x - 18$; $[2, 4]$



$\left\{\frac{9 + \sqrt{3}}{3}, \frac{9 - \sqrt{3}}{3}\right\}$

3) $y = -\frac{x^2}{2} + x - \frac{1}{2}$; $[-2, 1]$

$\left\{-\frac{1}{2}\right\}$

4) $y = \frac{x^2}{2} - 2x - 1$; $[-1, 1]$

$\{0\}$

5) $y = x^3 + 3x^2 - 2$; $[-2, 0]$

$\left\{\frac{-3 + \sqrt{3}}{3}, \frac{-3 - \sqrt{3}}{3}\right\}$

6) $y = -x^3 + 4x^2 - 3$; $[0, 4]$

$\left\{\frac{8}{3}\right\}$

7) $y = \frac{x^2 - 9}{3x}$; $[1, 4]$

$\{2\}$

8) $y = \frac{x^2}{2x - 4}$; $[-4, 1]$

$\{2 - \sqrt{6}\}$

9) $y = -(-2x + 6)^{\frac{1}{2}}$; $[-2, 3]$

10) $y = -(-5x + 25)^{\frac{1}{2}}$; $[3, 5]$

For each problem, determine if the Mean Value Theorem can be applied. If it can, find all values of *c* that satisfy the theorem. If it cannot, explain why not.

11) $y = -\frac{x^2}{4x + 8}$; $[-3, -1]$

12) $y = \frac{-x^2 + 9}{4x}$; $[1, 3]$

13) $y = -(6x + 24)^{\frac{2}{3}}$; $[-4, -1]$

14) $y = (x - 3)^{\frac{2}{3}}$; $[1, 4]$

Critical thinking question:

15) Use the Mean Value Theorem to prove that $|\sin a - \sin b| \le |a - b|$ for all real values of a and b where $a \ne b$.

Answers

9) $y = -(-2x + 6)^{\frac{1}{2}}$; $[-2, 3]$

$\left\{\frac{7}{4}\right\}$

10) $y = -(-5x + 25)^{\frac{1}{2}}$; $[3, 5]$

$\left\{\frac{9}{2}\right\}$

For each problem, determine if the Mean Value Theorem can be applied. If it can, find all values of *c* that satisfy the theorem. If it cannot, explain why not.

11) $y = -\frac{x^2}{4x + 8}$; $[-3, -1]$

The function is not continuous on $[-3, -1]$

12) $y = \frac{-x^2 + 9}{4x}$; $[1, 3]$

$\{\sqrt{3}\}$

13) $y = -(6x + 24)^{\frac{2}{3}}$; $[-4, -1]$

$\left\{-\frac{28}{9}\right\}$

14) $y = (x - 3)^{\frac{2}{3}}$; $[1, 4]$

The function is not differentiable on $(1, 4)$

Critical thinking question:

15) Use the Mean Value Theorem to prove that $|\sin a - \sin b| \le |a - b|$ for all real values of a and b where $a \ne b$.

Let $f(x) = \sin x$. Use the interval $[a,b]$. By the MVT, we know that there is at least one c such that $\frac{\sin b - \sin a}{b - a} = \cos c$. We know $\cos c \le 1$ for all c. Therefore, $\frac{\sin b - \sin a}{b - a} \le 1$, $\frac{|\sin a - \sin b|}{|a - b|} \le 1$, and $|\sin a - \sin b| \le |a - b|$.