

Domain & Range of Functions

➤ The **domain** of a function is the set of all the numbers you can substitute into the function (x-values).

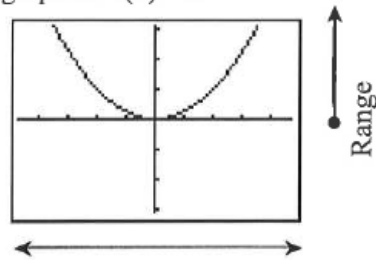
➤ The **range** of a function is the set of all the numbers you can get out of the function (y-values).

For example: $f(x) = x^2$

What's the domain? Well, you can substitute any real number you want into this function: You can square 4, $\frac{1}{2}$, -7, 1.01738, or whatever, and you get an answer. So the domain is all real numbers. In interval notation this is written $(-\infty, \infty)$.

What's the range? Well, let's think about it. If you substitute any number into this function, are you ever going to be able to get a negative number out of it? Nope! So it looks like the range of this function is the set of all non-negative numbers (the positive numbers plus zero). In interval notation this is written $[0, \infty)$.

Now look at the graph of $f(x) = x^2$



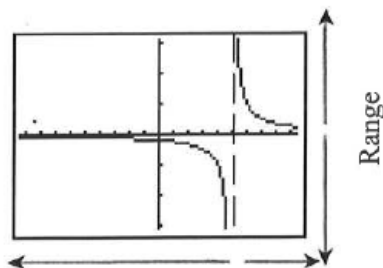
Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

Let's try another example: $f(x) = \frac{1}{x - 5}$

What numbers can we substitute into this function? Well, it looks like any number will do. But wait! What happens if we substitute in $x = 5$? We divide by zero, which is not allowed. So let's not substitute in 5. Since any other number is fine, the domain of this function is: all real numbers except 5. In interval notation this is written $(-\infty, 5) \cup (5, \infty)$.

Range is sometimes harder to figure out from a formula. This is where the graph comes in handy.



Domain: $(-\infty, 5) \cup (5, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$ [since the graph never touches the x-axis]

When determining the domain of a function from a formula, we really only have to look out for two situations:

(1) **Rational Expressions** (fractions) - Division by zero is not allowed so we must omit any values of x which make the denominator zero.

(2) **Even Roots** - For even roots such as square roots, the radicand can not be negative (the square root of a negative number is not defined as a real number). In this situation we must make sure the radicand is non-negative (that is, greater than or equal to zero).

Examples Find the domain of each of the following functions

$$(A) \quad f(x) = \frac{5x}{x^2 - 3x - 4}$$

Since this is a rational expression, we must not let the denominator equal zero. What values of x make the denominator zero?

$$x^2 - 3x - 4 = 0$$

$$(x - 4)(x + 1) = 0$$

$$x = 4 \quad \text{or} \quad x = -1$$

Thus all real numbers work except -1 and 4. In interval notation this is written:

$$(-\infty, -1) \cup (1, 4) \cup (4, \infty)$$

$$(B) \quad f(x) = \sqrt{2x + 3} - 1$$

Since this function involves a square root we must make sure the radicand is non-negative:

$$2x + 3 \geq 0$$

$$2x \geq -3$$

$$x \geq -3/2$$

Thus the domain is all real numbers greater than or equal to $-3/2$. In interval notation this is written: $[-3/2, \infty)$

$$(C) \quad f(x) = x^2 + 5x - 7$$

This function does not involve a rational expression or a square root so the domain is all real numbers. In interval notation this is written: $(-\infty, \infty)$

$$(D) \quad f(x) = \frac{2}{\sqrt{x - 1}}$$

We must be very careful with this function since it involves both a rational expression and a square root. The square root requires the radicand to be greater than or equal to zero, that is, $x - 1 \geq 0$. However, since the square root is in the denominator and we can not divide by zero, we cannot let $x - 1 = 0$. Thus,

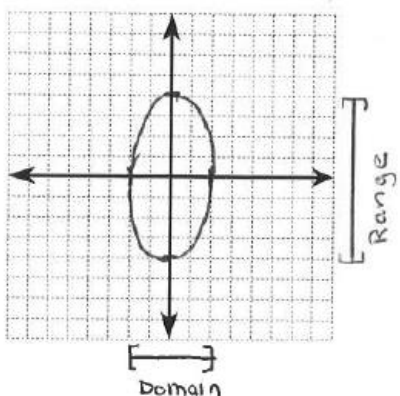
$$x - 1 > 0$$

$$x > 1$$

In interval notation this is written: $(1, \infty)$

Now let's look at some graphs and determine their domain and range.

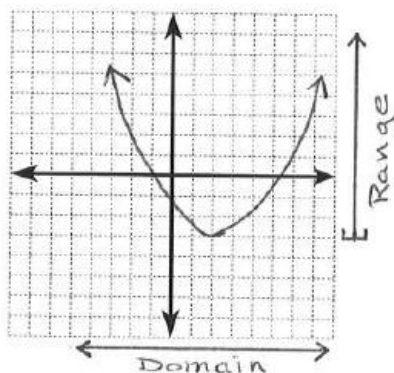
(A)



From the graph we see the x-values extend from -2 to 2 inclusive, so
Domain: $[-2, 2]$

The y-values extend from -4 to 4 inclusive, so
Range: $[-4, 4]$

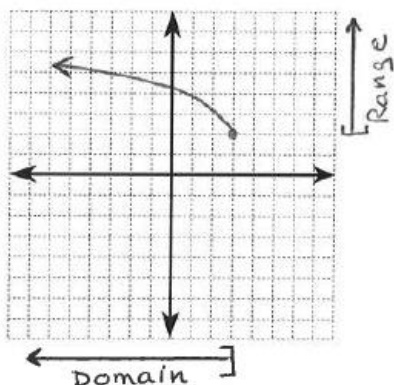
(B)



From the graph we see the x-values extend indefinitely in both directions, so
Domain: $(-\infty, \infty)$

The y-values start at -3 and keep going up, so,
Range: $[-3, \infty)$

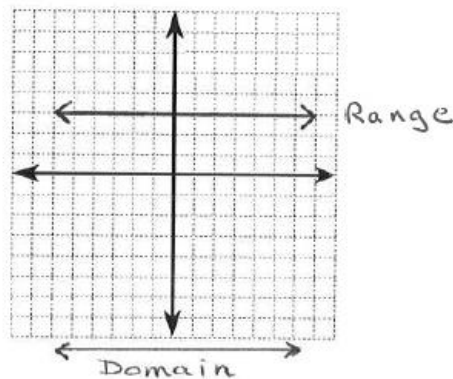
(C)



From the graph we see the x-values extend indefinitely on the left, but stop at 3, so
Domain: $(-\infty, 3]$

The y-values start at 2 and keep going up, so
Range: $[2, \infty)$

(D)



From the graph we see the x-values extend indefinitely in both directions, so
Domain: $(-\infty, \infty)$

The only y-value is 3, so
Range: $\{3\}$ notice this is not an interval

Determine the domain and range of the relation.

1) $\{(11, -6), (-9, 4), (1, 3), (-1, 2), (8, -5)\}$

Find the domain of the function.

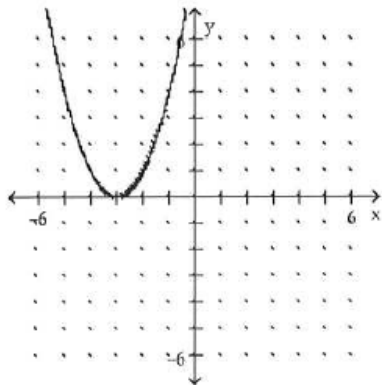
2) $f(x) = 5x^2 + 3x - 1$

3) $f(x) = \sqrt{x-6} + 2$

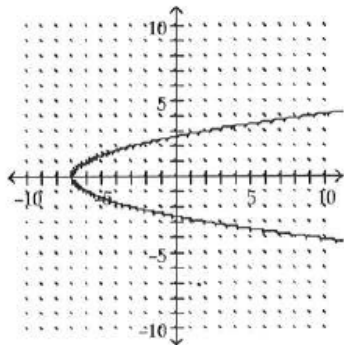
4) $f(x) = \frac{1}{x^2 + 3x - 10}$

Find the domain and range of the relation. Assume the ends of the relation continue on.

5)



6)



Answers

1) Answer: $D = \{8, 11, 1, -9, -1\}$; $R = \{-5, -6, 3, 4, 2\}$

2) Answer: $(-\infty, \infty)$

3) Answer: $[6, \infty)$

4) Answer: $(-\infty, -5) \cup (-5, 2) \cup (2, \infty)$

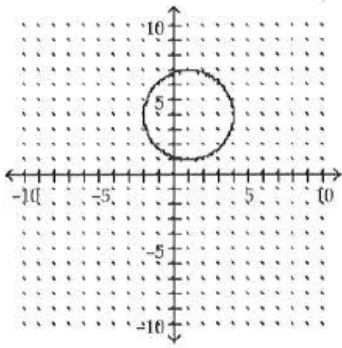
5) Answer: domain $(-\infty, \infty)$; range $[0, \infty)$

6) Answer: domain $[-7, \infty)$; range $(-\infty, \infty)$

7) Answer: domain $[-2, 4]$; range $[1, 7]$

8) Answer: domain $(-\infty, \infty)$; range $[-3, \infty)$

7)



8)

