Matrix Basic Operations … Set 1

Matrices

- an $m \times n$ matrix has m rows and n columns.
- each element is identified as a_{ij} where i is the row # and j is the column #.

$$\begin{array}{r}\text{row 1}\\ \text{row 2}\\ \text{row 3}\\ \text{etc.}\end{array}\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots \\ a_{21} & a_{22} & a_{23} & \cdots \\ a_{31} & a_{32} & a_{33} & \cdots \\ \vdots & \vdots & \vdots & \end{bmatrix}$$

$$\begin{array}{cccc} \uparrow & \uparrow & \uparrow & \\ \text{column 1} & \text{column 2} & \text{column 3} & \text{etc.} \end{array}$$

a 1×1 matrix: $\begin{bmatrix} 3 \end{bmatrix}$

a 1×2 matrix: $\begin{bmatrix} -1 & 4 \end{bmatrix}$

a 2×1 matrix: $\begin{bmatrix} 0 \\ 5 \end{bmatrix}$

a 2×2 matrix: $\begin{bmatrix} -3 & 0 \\ 1 & -2 \end{bmatrix}$

a 3×2 matrix: $\begin{bmatrix} 1 & 8 \\ 2 & 9 \\ 4 & 3 \end{bmatrix}$

a 1×3 matrix: $\begin{bmatrix} 10 & 11 & 12 \end{bmatrix}$

* the elements can be anything

* matrices can be combined through addition when they have the same $m \times n$ dimensions

* you can also multiply certain matrices together

* matrices can be multiplied by a constant

Matrix Basic Operations … Set 1

- Adding matrices of the same dimension

Ex.1) $\begin{bmatrix} 7 & 0 \\ -2 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 1 \\ 8 & 9 \end{bmatrix} = \begin{bmatrix} 7+5 & 0+1 \\ -2+8 & 4+9 \end{bmatrix} = \begin{bmatrix} 12 & 1 \\ 6 & 13 \end{bmatrix}$

*you add the elements from each matrix that are in the same a_{ij} position.

Ex.2) $\begin{bmatrix} 3 & 6 & -2 \\ 9 & 0 & 1 \\ 7 & -4 & 3 \end{bmatrix} + \begin{bmatrix} -4 & 6 & -1 \\ 2 & 8 & 0 \\ 0 & -2 & 5 \end{bmatrix} = \begin{bmatrix} -1 & 12 & -3 \\ 11 & 8 & 1 \\ 7 & -6 & 8 \end{bmatrix}$

- Multiplyin matrices by a constant.

Ex.1) $4\begin{bmatrix} 2 & 5 \\ -3 & 0 \\ 1 & 8 \end{bmatrix} = \begin{bmatrix} 4\cdot 2 & 4\cdot 5 \\ 4\cdot -3 & 4\cdot 0 \\ 4\cdot 1 & 4\cdot 8 \end{bmatrix} = \begin{bmatrix} 8 & 20 \\ -12 & 0 \\ 4 & 32 \end{bmatrix}$

* you multiply each element by the constant.

Ex.2) $-3\begin{bmatrix} -1 & 2 & 1 \\ 3 & 4 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -6 & -3 \\ -9 & -12 & 6 \end{bmatrix}$

• Multiplying matrices together

* matrices can only be multiplied together when their dimensions are in the form $m \times n \cdot n \times m$

** so the left matrix's # of columns should match the right matrix's # of rows. THIS IS THE EXACT WAY TO DETERMINE IF YOU CAN MULTIPLY 2 MATRICES!!!

You can multiply:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} e & f & g \\ h & i & j \end{bmatrix}$$

2×2 ✓ 2×3

but you can't multiply:

$$\begin{bmatrix} e & f & g \\ h & i & j \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

2×3 ✗ 2×2

Ex.1) $\begin{bmatrix} 1 & 4 \\ 5 & 3 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 & 0 \\ -2 & -1 & 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} = \begin{bmatrix} -6 & -1 & 4 \\ 4 & 12 & -3 \end{bmatrix}$

2×2 ✓ 2×3

* check that you can multiply them
* if yes, then the resulting matrix will be a 2×3.

so the "outside" dimensions are for the new matrix if the "inside" dimensions match

• For the new matrix elements:

a_{11}: take (row 1)·(column 1) and add

$\begin{bmatrix} 1 & 4 \\ & \end{bmatrix} \cdot \begin{bmatrix} 2 & & \\ -2 & & \end{bmatrix}$ $a_{11} = 1(2) + 4(-2) = -6$

a_{21}: take (row 2)·(coulumn 1) and add

$\begin{bmatrix} & \\ 5 & 3 \end{bmatrix} \cdot \begin{bmatrix} 2 & & \\ -2 & & \end{bmatrix}$ $a_{21} = 5(2) + 3(-2) = 4$

$a_{12}: 1(3) + 4(-1) = -1$ $\qquad a_{13}: 1(0) + 4(1) = 4$

$a_{22}: 5(3) + 3(-1) = 12$ $\qquad a_{23}: 5(0) + 3(-1) = -3$

Determinants

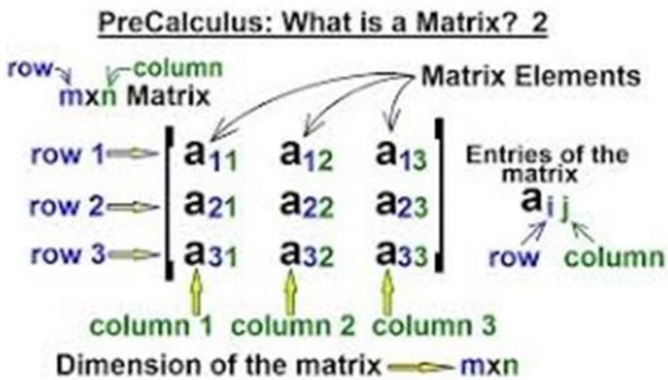
- For any 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the <u>determinant</u> is $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = a\cdot d - b\cdot c$.

Ex.1) Evaluate $\begin{vmatrix} 7 & 4 \\ 3 & 2 \end{vmatrix} = (7)(2) - (4)(3) = 14 - 12$
$= 2$

Ex.2) Evaluate the determinant of $\begin{bmatrix} 4 & 2 \\ 8 & 6 \end{bmatrix}$.

$\begin{vmatrix} 4 & 2 \\ 8 & 6 \end{vmatrix} = (4)(6) - (2)(8) = 24 - 16 = 8.$

Matrix Basic Operations … Set 1



Matrix Basic Operations … Set 1

Basic Matrix Operations

Simplify. Write "undefined" for expressions that are undefined. *can't be combined due to dimensions.

1) $\begin{bmatrix} 3 & 6 \\ -1 & -3 \\ -5 & -1 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 6 & 0 \\ 2 & 3 \end{bmatrix}$ ✓

$= \begin{bmatrix} 3+0 & 6-1 \\ -1+6 & -3+0 \\ -5+2 & -1+3 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 5 & -3 \\ -3 & 2 \end{bmatrix}$

2) $\begin{bmatrix} -5 & 2 & -2 \\ 4 & -2 & 0 \end{bmatrix} - \begin{bmatrix} 6 & -5 & -6 \\ 1 & 3 & -3 \end{bmatrix}$

3) $-5\begin{bmatrix} 5 & 6 & -4 \\ 4 & -2 & -1 \end{bmatrix}$ each element multiplied by −5

$= \begin{bmatrix} -25 & -30 & 20 \\ -20 & 10 & 5 \end{bmatrix}$

4) $-5\begin{bmatrix} -3 & 0 \\ 0 & 5 \end{bmatrix}$

5) $\begin{bmatrix} 4 & 2 \end{bmatrix} + \begin{bmatrix} -2 & -6 \end{bmatrix}$

$= \begin{bmatrix} 4-2 & 2-6 \end{bmatrix}$

$= \begin{bmatrix} 2 & -4 \end{bmatrix}$

6) $5\begin{bmatrix} 4 \\ 3 \end{bmatrix}$

7) $-5\begin{bmatrix} 1 & -2 & -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} -5 & 10 & -5 & -10 \end{bmatrix}$

8) $5\begin{bmatrix} 5 & 1 \\ 1 & -2 \\ 1 & 2 \end{bmatrix}$

9) $-2u\begin{bmatrix} 7u & 3w^2 & 5u & 5 \end{bmatrix}$

$= \begin{bmatrix} -14u^2 & -6uw^2 & -10u^2 & -10u \end{bmatrix}$

10) $\begin{bmatrix} 2 \\ 4 \end{bmatrix} + \begin{bmatrix} 5 \\ 6 \end{bmatrix}$

11) $4\begin{bmatrix} -4 \\ 3 \\ -5 \end{bmatrix}$

$= \begin{bmatrix} -16 \\ 12 \\ -20 \end{bmatrix}$

12) $\begin{bmatrix} -4n & n+m \\ -2n & -4n \end{bmatrix} + \begin{bmatrix} 4 & -5 \\ 3m & 0 \end{bmatrix}$

Matrix Basic Operations … Set 1

Basic Matrix Operations

*can't be combined due to dimensions

Simplify. Write "undefined" for expressions that are undefined.

✓ 1) $\begin{bmatrix} 3 & 6 \\ -1 & -3 \\ -5 & -1 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 6 & 0 \\ 2 & 3 \end{bmatrix}$

$= \begin{bmatrix} 3+0 & 6-1 \\ -1+6 & -3+0 \\ -5+2 & -1+3 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 5 & -3 \\ -3 & 2 \end{bmatrix}$

2) $\begin{bmatrix} -5 & 2 & -2 \\ 4 & -2 & 0 \end{bmatrix} - \begin{bmatrix} 6 & -5 & -6 \\ 1 & 3 & -3 \end{bmatrix}$

3) $-5\begin{bmatrix} 5 & 6 & -4 \\ 4 & -2 & -1 \end{bmatrix}$ each element multiplied by -5

$= \begin{bmatrix} -25 & -30 & 20 \\ -20 & 10 & 5 \end{bmatrix}$

4) $-5\begin{bmatrix} -3 & 0 \\ 0 & 5 \end{bmatrix}$

5) $\begin{bmatrix} 4 & 2 \end{bmatrix} + \begin{bmatrix} -2 & -6 \end{bmatrix}$

$= \begin{bmatrix} 4-2 & 2-6 \end{bmatrix}$

$= \begin{bmatrix} 2 & -4 \end{bmatrix}$

6) $5\begin{bmatrix} 4 \\ 3 \end{bmatrix}$

7) $-5\begin{bmatrix} 1 & -2 & -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} -5 & 10 & -5 & -10 \end{bmatrix}$

8) $5\begin{bmatrix} 5 & 1 \\ 1 & -2 \\ 1 & 2 \end{bmatrix}$

9) $-2u\begin{bmatrix} 7u & 3w^2 & 5u & 5 \end{bmatrix}$

$= \begin{bmatrix} -14u^2 & -6uw^2 & -10u^2 & -10u \end{bmatrix}$

10) $\begin{bmatrix} 2 \\ 4 \end{bmatrix} + \begin{bmatrix} 5 \\ 6 \end{bmatrix}$

11) $4\begin{bmatrix} -4 \\ 3 \\ -5 \end{bmatrix}$

$= \begin{bmatrix} -16 \\ 12 \\ -20 \end{bmatrix}$

12) $\begin{bmatrix} -4n & n+m \\ -2n & -4n \end{bmatrix} + \begin{bmatrix} 4 & -5 \\ 3m & 0 \end{bmatrix}$

Matrix Basic Operations … Set 1

All Matrix Operations

can't be combined due to size

Simplify. Write "undefined" for expressions that are $\boxed{\text{undefined.}}$

1) $\begin{array}{c} \text{rows} \\ \begin{bmatrix} 2 & -1 \\ -6 & 1 \end{bmatrix} \end{array} \cdot \begin{array}{c} \text{columns} \\ \begin{bmatrix} 4 & 4 \\ -3 & -5 \end{bmatrix} \end{array} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$2\times 2 \cdot 2\times 2 \checkmark = 2\times 2$

$a_{\underline{11}}: 2(4) + -1(-3) = 11$
row 1 and column 1
$a_{\underline{21}}: -6(4) + 1(-3) = -27$
row 2 and column 1
etc.

$= \begin{bmatrix} 11 & 13 \\ -27 & -29 \end{bmatrix}$

2) $\begin{bmatrix} 2 & 6 \\ -6 & 4 \end{bmatrix} \cdot \left(\begin{bmatrix} 5 & 3 \\ -6 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix} \right)$

3) $\begin{array}{c} 2\times 2 \\ \begin{bmatrix} -1 & 5 \\ 5 & -5 \end{bmatrix} \end{array} \cdot \begin{array}{c} 2\times 2 \\ \begin{bmatrix} -3 & 6 \\ -3 & 0 \end{bmatrix} \end{array} \checkmark \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

a_{11}: $-1(-3) + 5(-3) = -12$
a_{21}: $5(-3) + -5(-3) = 0$
a_{12}: $-1(6) + 5(0) = -6$
a_{22}: $5(6) + -5(0) = 30$

$= \begin{bmatrix} -12 & -6 \\ 0 & 30 \end{bmatrix}$

4) $\begin{bmatrix} 1 & -6 \\ 3 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 5 \end{bmatrix} + \begin{bmatrix} -3 \\ 0 \\ 3 \\ -2 \end{bmatrix}$

$m\times n \checkmark$

5) $\begin{bmatrix} -2 \\ -3 \\ -6 \\ 2 \end{bmatrix} + \begin{bmatrix} -4 \\ 6 \\ 0 \\ -3 \end{bmatrix}$

$= \begin{bmatrix} -2+-4 \\ -3+6 \\ -6+0 \\ 2+-3 \end{bmatrix} = \begin{bmatrix} -6 \\ 3 \\ -6 \\ -1 \end{bmatrix}$

6) $-4 \cdot \left(\begin{bmatrix} -3 & -6 \\ 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} -2 & 6 \\ -1 & -4 \end{bmatrix} \right)$

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7) $\begin{bmatrix} 3 & 1 & 3 \\ 0 & 5 & -3 \end{bmatrix} + \boxed{\begin{bmatrix} -1 & 3 \\ 6 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & -6 & -1 \\ 1 & 1 & 4 \end{bmatrix}}$ 1st

$2\times 2 \checkmark 2\times 3 \checkmark$
$= 2\times 3$

$= \begin{bmatrix} 3 & 1 & 3 \\ 0 & 5 & -3 \end{bmatrix} + \begin{bmatrix} 3 & 9 & 13 \\ 1 & -35 & -2 \end{bmatrix}$

$= \begin{bmatrix} 6 & 10 & 16 \\ 1 & -30 & -5 \end{bmatrix}$

8) $-5\left(\begin{bmatrix} 2 \\ -1 \\ -6 \end{bmatrix} + \begin{bmatrix} 2 \\ -4 \\ 4 \end{bmatrix} \right)$

Matrix Multiplication

Simplify. Write "undefined" for expressions that are undefined.

✓ 2x2 2x2 = 2x2

1) $\begin{bmatrix} 0 & 2 \\ -2 & -5 \end{bmatrix} \cdot \begin{bmatrix} 6 & -6 \\ 3 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0\cdot 6 + 2\cdot 3 & 0\cdot -6 + 2\cdot 0 \\ -2\cdot 6 + -5\cdot 3 & -2\cdot -6 + -5\cdot 0 \end{bmatrix}$

$= \begin{bmatrix} 6 & 0 \\ -27 & 12 \end{bmatrix}$

2) $\begin{bmatrix} 6 \\ -3 \end{bmatrix} \cdot \begin{bmatrix} -5 & 4 \end{bmatrix}$

✓ 2x2 2x2 = 2x2

3) $\begin{bmatrix} -5 & -5 \\ -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} -2 & -3 \\ 3 & 5 \end{bmatrix}$

$= \begin{bmatrix} -5 & -10 \\ 8 & 13 \end{bmatrix}$

4) $\begin{bmatrix} -3 & 5 \\ -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 6 & -2 \\ 1 & -5 \end{bmatrix}$

✓ 3x2 2x2 = 3x2

5) $\begin{bmatrix} 0 & 5 \\ -3 & 1 \\ -5 & 1 \end{bmatrix} \cdot \begin{bmatrix} -4 & 4 \\ -2 & -4 \end{bmatrix}$

$= \begin{bmatrix} -10 & -20 \\ 10 & -16 \\ 18 & -24 \end{bmatrix}$

6) $\begin{bmatrix} 5 & 3 & 5 \\ 1 & 5 & 0 \end{bmatrix} \cdot \begin{bmatrix} -4 & 2 \\ -3 & 4 \\ 3 & -5 \end{bmatrix}$

✓ 3x1 1x2 = 3x2

7) $\begin{bmatrix} -5 \\ 6 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 3 & -1 \end{bmatrix}$

$= \begin{bmatrix} -5\cdot 3 & -5\cdot -1 \\ 6\cdot 3 & 6\cdot -1 \\ 0\cdot 3 & 0\cdot -1 \end{bmatrix} = \begin{bmatrix} -15 & 5 \\ 18 & -6 \\ 0 & 0 \end{bmatrix}$

8) $\begin{bmatrix} 3 & 2 & 5 \\ 2 & 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 4 & 5 & -5 \\ 5 & -1 & 6 \end{bmatrix}$

Matrix Basic Operations … Set 1

$\begin{vmatrix} a & b \\ c & d \end{vmatrix}$

Determinants of 2×2 Matrices $= ad - bc$

Evaluate the determinant of each matrix.

1) $\begin{bmatrix} 0 & -4 \\ -6 & -2 \end{bmatrix}$ $\begin{vmatrix} 0 & -4 \\ -6 & -2 \end{vmatrix} =$

$(0)(-2) + (-4)(-6) = 24$

2) $\begin{bmatrix} -6 & 0 \\ 6 & -6 \end{bmatrix}$

3) $\begin{bmatrix} -1 & 1 \\ -1 & 4 \end{bmatrix}$ $\begin{vmatrix} -1 & 1 \\ -1 & 4 \end{vmatrix} =$

$(-1)(4) + (1)(-1) = -5$

4) $\begin{bmatrix} 0 & 4 \\ 6 & 5 \end{bmatrix}$

5) $\begin{bmatrix} 0 & -1 \\ 6 & -6 \end{bmatrix}$ $\begin{vmatrix} 0 & -1 \\ 6 & -6 \end{vmatrix} =$

$(0)(-6) + (-1)(6) = -6$

6) $\begin{bmatrix} 5 & 3 \\ 6 & 6 \end{bmatrix}$

Evaluate each determinant.

7) $\begin{vmatrix} -5 & 3 \\ 4 & 2 \end{vmatrix}$

$(-5)(2) + (3)(4) = 2$

8) $\begin{vmatrix} -9 & -9 \\ -7 & -10 \end{vmatrix}$

9) $\begin{vmatrix} -1 & 8 \\ 5 & 0 \end{vmatrix}$

$(-1)(0) + (8)(5) = 40$

10) $\begin{vmatrix} 8 & -6 \\ -10 & 9 \end{vmatrix}$

11) $\begin{vmatrix} 0 & 6 \\ -8 & 0 \end{vmatrix}$

$(0)(0) + (6)(-8) = -48$

12) $\begin{vmatrix} 10 & -9 \\ -7 & 3 \end{vmatrix}$

13) $\begin{vmatrix} -5 & 0 \\ 2 & 10 \end{vmatrix}$

$(-5)(10) + (0)(2) = -50$

14) $\begin{vmatrix} 2 & -2 \\ 7 & -7 \end{vmatrix}$

15) Evaluate:

$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} + \begin{vmatrix} 5 & 2 \\ -2 & 6 \end{vmatrix}$

$(1)(4) + (2)(3) + (5)(6) + (2)(-2)$

$10 + 26$

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16) Give an example of a 2×2 matrix whose determinant is 13.