

Definition of Derivative

Derivatives

Definition and Notation

If $y = f(x)$ then the derivative is defined to be $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$.

If $y = f(x)$ then all of the following are equivalent notations for the derivative.

$$f'(x) = y' = \frac{df}{dx} = \frac{dy}{dx} = \frac{d}{dx}(f(x)) = Df(x)$$

If $y = f(x)$ all of the following are equivalent notations for derivative evaluated at $x = a$.

$$f'(a) = y'|_{x=a} = \left. \frac{df}{dx} \right|_{x=a} = \left. \frac{dy}{dx} \right|_{x=a} = Df(a)$$

Interpretation of the Derivative

If $y = f(x)$ then,

1. $m = f'(a)$ is the slope of the tangent line to $y = f(x)$ at $x = a$ and the equation of the tangent line at $x = a$ is given by $y = f(a) + f'(a)(x - a)$.

2. $f'(a)$ is the instantaneous rate of change of $f(x)$ at $x = a$.

3. If $f(x)$ is the position of an object at time x then $f'(a)$ is the velocity of the object at $x = a$.