Systems of Equations and Matrices … Set 1

Matrices and Systems of Equations

State the dimensions of each matrix.

1) $\begin{bmatrix} 1 \end{bmatrix}$

This matrix has_____ row and _____ columns.
The dimension of the matrix is _____ × _____.

2) $\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}$

This matrix has_____ row and _____ columns.
The dimension of the matrix is _____ × _____.

3) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

This matrix has_____ row and _____ columns.
The dimension of the matrix is _____ × _____.

4) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$

This matrix has_____ row and _____ columns.
The dimension of the matrix is _____ × _____.

5) $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$

This matrix has_____ row and _____ columns.
The dimension of the matrix is _____ × _____.

6) $\begin{bmatrix} 1 & 5 & 9 \\ 2 & 6 & 10 \\ 3 & 7 & 11 \\ 4 & 8 & 12 \end{bmatrix}$

This matrix has_____ row and _____ columns.
The dimension of the matrix is _____ × _____.

Examples of Matrices

7) Row Matrix: Column Matrix:

   System: Augmented Matrix:

   Coefficient Matrix: Constant Matrix:

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Writing an Augmented Matrix: Write the augmented matrix for the system of equations.

8) $x + 3y = 9$
   $-y + 4z = -2$
   $x - 5z = 0$

9) $x + y + z = 2$
   $2x - y + 3z = -1$
   $-x + 2y - z = 4$

10) $x + 3z = 2$
    $-2y + z = -5$
    $6x + 4y = 7$

11) $-3y - z = 1$
    $2x + 3y + 9z = 0$
    $5x - 2z = -3$

Elementary Row Operations

12) Interchange the first and second rows.
$$\begin{bmatrix} 0 & 1 & 3 & 4 \\ -1 & 2 & 0 & 3 \\ 2 & -3 & 4 & 1 \end{bmatrix}$$

13) Interchange the first and third rows.
$$\begin{bmatrix} 1 & 3 & 4 & 0 \\ 0 & 3 & -1 & 2 \\ 1 & 2 & -3 & 4 \end{bmatrix}$$

14) Multiply the first row by $\dfrac{1}{2}$.
$$\begin{bmatrix} 2 & -4 & 6 & -2 \\ 1 & 3 & -3 & 0 \\ 5 & -2 & 1 & 2 \end{bmatrix}$$

15) Multiply the second row by $\dfrac{1}{2}$.
$$\begin{bmatrix} 1 & 0 & 3 & 6 \\ 2 & -8 & 4 & 0 \\ 5 & 7 & 9 & 4 \end{bmatrix}$$

16) Add −2 times the first row of the matrix to the third row.
$$\begin{bmatrix} 1 & 2 & -4 & 3 \\ 0 & 3 & -2 & -1 \\ 2 & 1 & 5 & -2 \end{bmatrix}$$

17) Add −3 times the first row of the matrix to the second row.
$$\begin{bmatrix} 14 & 5 & 9 & -1 \\ 0 & 5 & 2 & 3 \\ -2 & 7 & -3 & 1 \end{bmatrix}$$

Comparing Linear Systems with Matrix Operations: Solve each system.

18) $x - 2y + 3z = 9$
    $-x + 3y + z = -2$
    $2x - 5y + 5z = 17$

19) $x + 3y + 4z = 7$
    $2x + 7y + 5z = 10$
    $3x + 10y + 4z = 27$

20) $2x + 4y + z = 1$
    $x - 2y - 3z = 2$
    $x + y - z = -1$

21) $x + y + z = 6$
    $2x - y + z = 3$
    $3x + y - z = 2$

Row-Echelon Form

A matrix in row-echelon form has the following properties:
1. Any rows consisting entirely of zeros occur at the bottom of the matrix.
2. For each row that does not consist entirely of zeros, the first nonzero entry is 1. It's called a leading 1.
3. For two successive (nonzero) rows, the leading 1 in the higher row is farther to the left than the leading 1 in the lower row.

A matrix is row-echelon form is in reduced row-echelon form when every column that has a leading 1 has zeros in every position above and below its leading 1.

Determine whether each matrix is in row-echelon form. If it is, determine whether the matrix is in reduced row-echelon form.

22) $\begin{bmatrix} 1 & 2 & -1 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -2 \end{bmatrix}$

23) $\begin{bmatrix} 1 & 2 & -1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & -4 \end{bmatrix}$

24) $\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

25) $\begin{bmatrix} 1 & -5 & 2 & -1 & 3 \\ 0 & 0 & 1 & 3 & -2 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

26) $\begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & 2 & 1 & -1 \\ 0 & 0 & 1 & -3 \end{bmatrix}$

27) $\begin{bmatrix} 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Gaussian Elimination with Back-Substitution: Solve each system of equations.

28) $y + z - 2w = -3$
$x + 2y - z = 2$
$2x + 4y + z - 3w = -2$
$x - 4y - 7z - w = -19$

29) $2x - y + z = 7$
$x + y - z + 3w = -4$
$x - 4y + 3z - 2w = 18$
$y + z - w = 1$

30) $3x + 2y - z + w = 0$
$x - y + 4z + 2w = 25$
$-2x + y + 2z - w = 2$
$x + y + z + w = 6$

31) $x - 4y + 3z - 2w = 9$
$3x - 2y + z - 4w = -13$
$-4x + 3y - 2z + w = -4$
$-2x + y - 4z + 3w = -10$

Gauss-Jordan Elimination: Use Gauss-Jordan elimination to solve each system.

32) $x - 2y + 3z = 9$
$-x + 3y + z = -2$
$2x - 5y + 5z = 17$

33) $3x - 3y - 3z = 6$
$5x + 3y + z = 18$
$3x - y + z = 8$

34) $-x + 2y - 6z = 12$
$-x + 2y + 6z = -24$
$-x + 6y - 4z = -18$

35) $y + 2z = 6$
$-2x - y - z = -5$
$3x - 6y + 2z = 9$

36) $-6x - 2y + 6z = -8$
$3x + 2y + z = -10$
$5x + 6y + 2z = -23$

37) $-5x + 4y + 3z = -4$
$6x + 2y - 4z = -10$
$5x - 3y + 5z = 26$

38) $3x - 4y + 2z = -5$
$5x - 4y + 2z = -3$
$-3x + 6y - 3z = 9$

39) $5x - 4y - 3z = -13$
$2x + y + 3z = 3$
$-3x + 5y + 6z = 3$

40) $-2x - 3y - z = 21$
$-2x + 2y + 4z = 2$
$2x - 3y - 5z = 10$

41) $4x + y - 6z = -21$
$2x - 3y - 4z = -29$
$3x - y - 5z = -25$