Logarithms

Definition: $y = \log_a x$ if and only if $x = a^y$, where $a > 0$.
In other words, logarithms are exponents.

Remarks:

- $\log x$ always refers to log base 10, i.e., $\log x = \log_{10} x$.

- $\ln x$ is called the natural logarithm and is used to represent $\log_e x$, where the irrational number $e \approx 2.71828$. Therefore, $\ln x = y$ if and only if $e^y = x$.

- Most calculators can directly compute logs base 10 and the natural log. For any other base it is necessary to use the change of base formula: $\log_b a = \dfrac{\ln a}{\ln b}$ or $\dfrac{\log_{10} a}{\log_{10} b}$.

Properties of Logarithms (Recall that logs are only defined for positive values of x.)

For the natural logarithm	For logarithms base a
1. $\ln xy = \ln x + \ln y$	1. $\log_a xy = \log_a x + \log_a y$
2. $\ln \dfrac{x}{y} = \ln x - \ln y$	2. $\log_a \dfrac{x}{y} = \log_a x - \log_a y$
3. $\ln x^y = y \cdot \ln x$	3. $\log_a x^y = y \cdot \log_a x$
4. $\ln e^x = x$	4. $\log_a a^x = x$
5. $e^{\ln x} = x$	5. $a^{\log_a x} = x$

Useful Identities for Logarithms

For the natural logarithm	For logarithms base a
1. $\ln e = 1$	1. $\log_a a = 1$, for all $a > 0$
2. $\ln 1 = 0$	2. $\log_a 1 = 0$, for all $a > 0$