Sequences … Set 1

Introduction to sequences

For each sequence, find the next 4 terms. Describe the pattern in words.

1.   1, 2, 4, 7, 11, …

2.   3, 9, 27, 81, …

3.   1, 3, 7, 15, 31, …

4.   192, -96, 48, -24, …

5.   2, 6, 12, 20, 30, …

6. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

7. $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \ldots$

The following are multiple choice questions. Find the next three terms of each sequence. Next to the problem, describe the pattern in words.

Describe the Pattern:

8.   4, −16, 64, −256, 1024, …
   A) −1024, 4096, −16384   B) −4096, −16384, −65536
   C) −4096, 16384, −65536   D) −1024, −4096, −16384

9.   1, 5, 25, 125, 625, …
   A) 6250, 31250, 156250   B) 3125, 15625, 78125
   C) $\frac{2}{3125}, \frac{2}{15625}, \frac{2}{78125}$ D) $\frac{2}{625}, \frac{2}{3125}, \frac{2}{15625}$

Describe the Pattern:

10. 4, 16, 36, 64, 100, ...

     A) 144, 196, 281   B) 144, 196, 256

     C) 164, 196, 307   D) 144, 196, 300

11. 4, −12, 36, −108, 324, ...

     A) −972, −2916, −8748   B) −972, 2916, −8748

     C) $-\dfrac{4}{243}, \dfrac{4}{729}, -\dfrac{4}{2187}$ D) $-\dfrac{4}{243}, -\dfrac{4}{729}, -\dfrac{4}{2187}$

12. 10, 6, 4, 3, $\dfrac{5}{2}$, ...

     A) $\dfrac{5}{2}, \dfrac{19}{9}, \dfrac{33}{16}$ B) $\dfrac{9}{4}, \dfrac{17}{8}, \dfrac{33}{16}$

     C) $4, \dfrac{19}{9}, \dfrac{33}{16}$ D) $4, \dfrac{16}{13}, \dfrac{15}{8}$

13. 1, 9, 25, 49, 81, ...

     A) 121, 169, 225   B) 121, 193, 225

     C) 140, 160, 225   D) 167, 160, 225

14. Suppose you drop a tennis ball from a height of 15 feet. After the ball hits the floor, it rebounds to 85% of its previous height. How high will the ball rebound after its third bounce? Round to the nearest tenth.

15. The table shows how the number of sit-ups Marla does each day has changed over time. At this rate, how many sit-ups will she do on Day 12? Explain your steps in solving this problem.

Day1	Day 2	Day 3	Day 4	Day 5
28	33	38	43	48

Sequences … Set 1

nth term of a sequence

Multiple Choice: Circle the correct answer.

Find the first 4 terms of the sequence given the explicit formula.

1) $a_n = n^2 + 3$

   A) 1, 6, 10, 23
   B) 4, 7, 12, 22
   C) 4, 7, 12, 19
   D) 4, 6, 10, 20

2) $a_n = \dfrac{15}{n+3}$

   A) $\dfrac{15}{4}$, 3, $\dfrac{5}{2}$, $\dfrac{15}{7}$

   B) 3, 3, $\dfrac{5}{4}$, $\dfrac{15}{8}$

   C) $\dfrac{15}{4}$, 3, $\dfrac{3}{2}$, $\dfrac{19}{6}$

   D) 3, −2, $\dfrac{3}{2}$, $\dfrac{19}{6}$

Write the explicit formula for each sequence.

3) 3, −6, 12, −24, 48, ...

   A) $a_n = 3 \cdot (-2)^{n-1}$

   B) $a_n = -3 \cdot (-3)^{n-1}$

   C) $a_n = -3 \cdot 3^{n-1}$

   D) $a_n = 3 \cdot (-3)^{n-1}$

4) 4, 9, 16, 25, ...

   A) $a_n = n^2 + 1$

   B) $a_n = \dfrac{n^3}{2n+1}$

   C) $a_n = (n+1)(n+1)$

   D) $a_n = n^2$

5.   The table shows the predicted growth of a particular bacteria after various numbers of hours. Write an explicit formula for the sequence of the number of bacteria.

Hours (n)	1	2	3	4	5
Number of Bacteria	19	38	57	76	95

   A. $a_n = 19n + 19$ C. $a_n = \dfrac{1}{19}n$

   B. $a_n = n + 19$ D. $a_n = 19n$

6. Use the explicit formulas below to find a_1, a_2 and a_{10} for each formula. Show your work.

    a. $a_n = n^3 + 4$ b. $a_n = -5n + 3$

    c. $a_n = \dfrac{n+1}{n^2 + 3}$ d. $a_n = 2^n$ (this is 2 raised to the n-power, not 2 times n.)

7. Consider the following sequences. Write the explicit formula for each.

a. 5, 6, 7, 8, 9, …… $a_n =$ _____

b. 4, 8, 12, 16, 20…… $a_n =$ _____

a. 4, 16, 64, 256, 1024…… $a_n =$ _____

Sequences … Set 1

Arithmetic and Geometric Sequences

1. Are the following sequences arithmetic, geometric, or neither? If they are arithmetic, state the value of d. If they are geometric, state r.

    a)   6, 12, 18, 24, ...   _____

    b)   6, 11, 17, ...   _____

    c)   2, 14, 98, 686, ...   _____

    d)   160, 80, 40, 20, ...   _____

    e)   -40, -25, -10, 5, ...   _____

    f)   7, -21, 63, -189, ...   _____

Find the next 3 terms of the sequence. Determine whether the sequence is arithmetic or geometric.

2) 21, 29, 37, 45, ...   3) −36, −27, −18, −9, ...

4) −1, 6, −36, 216, ...   5) $\dfrac{1}{3}, \dfrac{2}{9}, \dfrac{4}{27}, \dfrac{8}{81}, ...$

6. Find the first 4 terms of each sequence. Determine whether each is arithmetic, geometric or neither. If so, find the common difference or common ratio.

a. $a_n = 172 - 200n$

b. $a_n = 3 \cdot 2^{n-1}$

c. $a_n = (-5)^{n-1}$
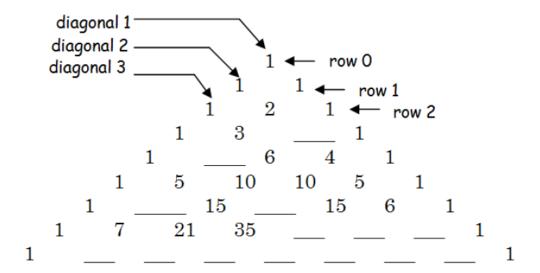
d. $a_n = 25 + 9n$

7. Karla opens a savings account at her bank with $500, compounded monthly with an annual rate of 2.5%

    a. How much is her monthly rate of interest?

    b. How much will she have in her account after 1 month?

    c. How much after 2 months?

        3 months?

    d. How much is in her account after 1-year?

    e. Is compound interest an arithmetic or a geometric sequence?

Sequences ... Set 1

Pascal's Triangle

```
diagonal 1
diagonal 2
diagonal 3                1   ←  row 0
                            1      1  ←  row 1
                          1    2     1  ←  row 2
                       1    3    ___   1
                    1    ___  6    4    1
                 1    5   10   10    5    1
              1   ___  15  ___   15   6    1
           1    7   21   35  ___ ___ ___   1
        1   ___ ___ ___ ___ ___ ___ ___   1
```

1. Fill in the blanks to Pascal's Triangle (There are blanks throughout the triangle!)

2. Find the sum of numbers in row 2 (see row labels) _____

3. Find the sum of the numbers in row 3 _____

4. Find the sum of the numbers in row 4 _____

5. Use the pattern from the previous 3 answers to find the sum of row 5 _____
Check your answer by adding the numbers in row 5.

6. Is the sum of numbers in the rows of Pascal's triangle an arithmetic or geometric sequence? Explain your answer.

7. Generate the next 6 terms of the Fibonacci Sequence.

    0, 1, 1, 2, 3...

8. **Bonus:** Find one application of the Fibonacci Sequence in nature.
   Print your finding and bring it to class. Make sure you can understand your example!

Sequences … Set 1

C. Which type of sequence is it??? Determine whether each the following is arithmetic, geometric or neither. Determine **d** or **r** where appropriate.

1. 0, 4, 8, 12....   arithmetic   geometric   neither   d = _____   r = _____

2. −7, -11, -15, -19....   arithmetic   geometric   neither   d = _____   r = _____

3. $\frac{1}{4}$, $\frac{1}{2}$, 1, 2....   arithmetic   geometric   neither   d = _____   r = _____

4. 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$...   arithmetic   geometric   neither   d = _____   r = _____

5. 3x, 5x, 7x, 9x....   arithmetic   geometric   neither   d = _____   r = _____

Given the explicit formula for a sequence find the first five terms.

6) $a_n = 2^n + 5$

Write the explicit formula for each sequence.

7) −1, 2, 7, 14, 23, ...

A) $a_n = -\dfrac{3}{n+2}$

B) $a_n = n^2 - 2$

C) $a_n = n^2 - 3$

D) $a_n = 10^n + 1$

8) 15, 23, 31, 39, 47, ...

A) $a_n = 21 - 6n$

B) $a_n = 22 - 7n$

C) $a_n = 7 + 8n$

D) $a_n = 9 + 6n$

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Sequences … Set 1

Recursion

Find the first four terms in each sequence.

1) $a_1 = -26$
$\qquad a_{n+1} = a_n + 6$

   A) $-22, -17, -12, -7$
   B) $-26, -21, -16, -11$
   C) $-24, -19, -14, -9$
   D) $-26, -20, -14, -8$

2) $a_1 = 2$
$\qquad a_{n+1} = a_n + 4$

   A) $5, 9, 13, 17$
   B) $0, 4, 8, 12$
   C) $2, 6, 10, 14$
   D) $1, 5, 9, 13$

3) $a_1 = 3$
$\qquad a_{n+1} = a_n \cdot 6$

   A) $3, -12, 48, -192$
   B) $3, \dfrac{1}{2}, \dfrac{1}{12}, \dfrac{1}{72}$
   C) $3, 18, 108, 648$
   D) $3, -15, 75, -375$

4) $a_1 = 1$
$\qquad a_{n+1} = a_n \cdot -2$

   A) $1, -2, 4, -8$
   B) $-\dfrac{1}{2}, 1, -2, 4$
   C) $-\dfrac{1}{2}, \dfrac{3}{2}, -\dfrac{9}{2}, \dfrac{27}{2}$
   D) $-\dfrac{3}{2}, \dfrac{9}{2}, -\dfrac{27}{2}, \dfrac{81}{2}$

5) $a_1 = 24$
$\qquad a_{n+1} = a_n + 20$

   A) $44, 24, 4, -16$
   B) $24, 44, 64, 84$
   C) $44, 64, 84, 104$
   D) $24, 48, 96, 192$

6) $a_1 = 26$
$\qquad a_{n+1} = \dfrac{2 + a_n}{2}$

   A) $26, 14, 8, 1$
   B) $26, 11, 8, 5$
   C) $26, 14, 8, 5$
   D) $26, 12, 8, 5$

Recursively Defined Sequences

Write the recursive rule in WORDS. Then, find the first 5 terms (the first one is already given, so you really only have to find the next 4. Write the first one anyway!)

1. $a_1 = 2$
$a_{n+1} = 4a_n$

2. $a_1 = -3$
$a_{n+1} = -2a_n$

3. $a_1 = 8$
$a_{n+1} = \dfrac{1}{2}a_n$

4. $a_1 = 3$
$a_{n+1} = a_n - 9$

5. $a_1 = 10$
$a_{n+1} = 2a_n + 4$

6. $a_1 = 18$
$a_{n+1} = \dfrac{2}{3}a_n + 18$

Series and Summation

Find the indicated sum.

1. 2, 4, 6, 8, 10...

$S_4 =$ _____

2. -1, 3, -5, 7, -9,...

$S_5 =$ _____

3. 4, 7, 10, 13, 16, ...

$S_2 =$ _____

4. -3, 9, -27, 81, -243, ...

$S_7 =$ _____

Evaluate each sum:

5. $\displaystyle\sum_{n=1}^{5} 2^n$

6. $\displaystyle\sum_{n=3}^{5} 4n + 3$

Evaluate each series.

7) $\displaystyle\sum_{n=4}^{8} (100 - n^2)$

A) 310   B) 120
C) 274   D) 329

8) $\displaystyle\sum_{m=1}^{6} \frac{120}{m}$

A) 274   B) $\dfrac{2178}{7}$

C) 294   D) 140

Rewrite each series using sigma notation.

9) $1 + 4 + 9 + 16 + 25 + 36$

A) $\displaystyle\sum_{a=1}^{4} a$ B) $\displaystyle\sum_{a=1}^{5} a$

C) $\displaystyle\sum_{a=3}^{9} a^2$ D) $\displaystyle\sum_{a=1}^{6} a^2$

10) $4 + 8 + 12 + 16 + 20$

A) $\displaystyle\sum_{a=1}^{3} a$ B) $\displaystyle\sum_{a=1}^{5} 2a$

C) $\displaystyle\sum_{a=1}^{5} 4a$ D) $\displaystyle\sum_{a=1}^{4} 5^a$

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Sequences … Set 1

Write the summation notation for the following:

11. $\dfrac{1}{2}+\dfrac{2}{3}+\dfrac{3}{4}+\dfrac{4}{5}+\dfrac{5}{6}+\dfrac{6}{7}$

12. $3+6+9+12+15$

13. $\dfrac{1}{1^2}+\dfrac{1}{2^2}+\dfrac{1}{3^2}+\dfrac{1}{4^2}+\dfrac{1}{5^2}$

14. Explain the significance of the $(-1)^{n+1}$ in the expression $\displaystyle\sum_{n=1}^{4}(-1)^{n+1}(4n+3)$.

_____
_____
_____

15. Express the series $3 - 5 + 7 - 9$ using sigma notation.