

FACTORING POLYNOMIALS

- 1) First determine if a common monomial factor (Greatest Common Factor) exists. Factor trees may be used to find the GCF of difficult numbers. Be aware of opposites: Ex. (a-b) and (b-a) These may become the same by factoring -1 from one of them.

$$\begin{aligned}3x - 12 &= 3(x - 4) \\x^2y^2 - 3xy^2 &= xy^2(x - 3) \\6(x - y) + a(x - y) &= (x - y)(6 + a)\end{aligned}$$

- 2) If the problem to be factored is a binomial, see if it fits one of the following situations.

- A. Difference of two squares:

$$\begin{aligned}a^2 - b^2 &= (a + b)(a - b) \\9x^2 - 25y^2 &= (3x + 5y)(3x - 5y) \\(a + b)^2 - 25 &= [(a + b) + 5][(a + b) - 5] = (a + b + 5)(a + b - 5)\end{aligned}$$

- B. Sum of two squares:

$$a^2 + b^2 \text{ does not factor (it is prime).}$$

- C. Sum of two cubes:

$$\begin{aligned}a^3 + b^3 &= (a + b)(a^2 - ab + b^2) \\8x^3 + 27y^3 &= (2x + 3y)(4x^2 - 6xy + 9y^2)\end{aligned}$$

Note: Resulting trinomial does not factor.

- D. Difference of two cubes:

$$\begin{aligned}a^3 - b^3 &= (a - b)(a^2 + ab + b^2) \\x^3 - 64 &= (x - 4)(x^2 + 4x + 16)\end{aligned}$$

Note: Resulting trinomial does not factor.

- E. If none of these occur, the binomial does not factor.

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3) If the problem is a trinomial, check for one of the following possibilities.

A. Square of a binomial:

$$\begin{aligned}a^2 + 2ab + b^2 &= (a + b)(a + b) = (a + b)^2 \\x^2 + 6x + 9 &= (x + 3)(x + 3) = (x + 3)^2 \\4x^2 - 20xy + 25y^2 &= (2x - 5y)^2\end{aligned}$$

B. If $a = 1$, use reverse foil or trial and error method:

$$\begin{aligned}x^2 + 7x + 12 &= (x + 3)(x + 4) \\x^2 - 7x + 12 &= (x - 3)(x - 4) \\x^2 + 3x - 18 &= (x + 6)(x - 3) \\x^2 - 3x - 18 &= (x - 6)(x + 3)\end{aligned}$$

C. If $a \neq 1$, use trial and error method. (Grouping may also be used.)

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4) If factoring a polynomial with four terms, possible choices are below.

A. Group first two terms together and last two terms together.

$$\begin{aligned}5a - 5b - xa + xb &= (5a - 5b) + (-xa + xb) = 5(a - b) - x(a - b) = (a - b)(5 - x) \\x^3 - 3x^2 + 2x - 6 &= (x^3 - 3x^2) + (2x - 6) = x^2(x - 3) + 2(x - 3) = (x - 3)(x^2 + 2)\end{aligned}$$

B. Group first three terms together.

$$x^2 + 6x + 9 - y^2 = (x^2 + 6x + 9) - y^2 = (x + 3)^2 - y^2 = [(x + 3) + y][(x + 3) - y] = (x + 3 + y)(x + 3 - y)$$

C. Group last three terms together.

$$y^2 - x^2 + 6x - 9 = y^2 - (x^2 - 6x + 9) = y^2 - (x - 3)^2 = [y + (x - 3)][y - (x - 3)] = (y + x - 3)(y - x + 3)$$

BE SURE YOUR ANSWERS WILL NOT FACTOR FURTHER!

All answers may be checked by multiplication.