Practice Test
Exponential Functions

1

If $f(x)=\sqrt{2x}$ and $g(x)=2x^2$, what is the value of $f(g(1))-g(f(1))$?

A) -4

B) -2

C) 2

D) 4

2

If $f(x)=\sqrt{625-x^2}$ and $g(x)=\sqrt{225-x^2}$, what is the value of $f(f(5))-g((g5))$?

A) 0

B) 5

C) 10

D) 20

3

The population of a certain town doubles every 25 years. If the population of the town was 51,200 in 1980, in what year was the population 6,400?

A) 1855

B) 1880

C) 1905

D) 1930

4

The half-life of a radioactive substance is the amount of time it takes for half of the substance to decay. The table below shows the time (in years) and the amount of substance left for a certain radioactive substance.

Time (years)	Amount (grams)
0	1,200
14	850
28	600
42	425
56	300

How much of the original amount of the substance to the nearest whole gram, will remain after 140 years?

A) 85

B) 75

C) 53

D) 38

5

A radioactive substance decays at a rate of 18% per year. If the initial amount of the substance is 100 grams, which of the following functions models the remaining amount of the substance, in grams, after t years?

A) $f(t)=100(0.18)^t$

B) $f(t)=100(0.82)^t$

C) $f(t)=100-100(0.18)^t$

D) $f(t)=100-100(0.82)^t$

6

$$5,000(1+\frac{r}{100})^t$$

The expression above gives the value of an investment, in dollars, that pays an annual interest rate of r% compounded yearly. 5,000 is the initial amount and t is the number of years after the initial amount was deposited. Which of the following expressions shows the difference between the value of a 15 year investment at 6% annual compound interest and a 12 year investment at 6% annual compound interest?

A) $5,000\left[(1.06)^{\frac{15}{12}}\right]$

B) $5,000\left[\frac{(1.06)^{15}}{(1.06)^{12}}\right]$

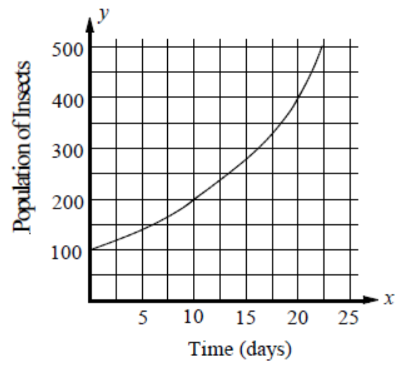
C) $5,000\left[(1.06)^{15}-(1.06)^{12}\right]$

D) $5,000\left[(1.06)^{15-12}\right]$

7

The price P, in dollars, of a truck t years after it was purchased is given by the function $P(t)=24,000(\frac{1}{2})^{\frac{t}{6}}$. To the nearest dollar, what is the price of the truck 9 years after it was purchased?

Questions 8 and 9 refer to the following information.



The graph above shows the size of a certain insect population over 25 days. The population at time $t=0$ was 100. A biologist used the equation $f(t)=100(2)^{\frac{t}{d}}$ to model the population.

8

What is the value of d in the equation?

9

What was the population of the insect after 15 days, to the nearest whole number?

Answers
Exponential Functions

1. B

$f(x) = \sqrt{2x}$ and $g(x) = 2x^2$

$g(1) = 2(1)^2 = 2$ and $f(1) = \sqrt{2(1)} = \sqrt{2}$

$f(g(1)) - g(f(1))$

$= f(2) - g(\sqrt{2})$

$= \sqrt{2(2)} - 2(\sqrt{2})^2$

$= \sqrt{4} - 2(2) = 2 - 4 = -2$

2. A

$f(x) = \sqrt{625 - x^2}$ and $g(x) = \sqrt{225 - x^2}$

$f(5) = \sqrt{625 - 5^2} = \sqrt{600}$

$g(5) = \sqrt{225 - 5^2} = \sqrt{200}$

$f(f(5)) - g((g5))$

$= f(\sqrt{600}) - g(\sqrt{200})$

$= (\sqrt{625 - (\sqrt{600})^2}) - (\sqrt{225 - (\sqrt{200})^2})$

$= \sqrt{625 - 600} - \sqrt{225 - 200}$

$= \sqrt{25} - \sqrt{25} = 0$

3. C

Method I:

You can keep dividing by 2 until you get to a population of 6,400.

Year	Population
1980	51,200
1955	25,600
1930	12,800
1905	6,400

Method II:

Use the half-life formula, $A = P(\frac{1}{2})^{t/d}$.

$6,400 = 51,200(\frac{1}{2})^{t/25}$

$\frac{6,400}{51,200} = (\frac{1}{2})^{t/25}$ Divide each side by 51,200.

$\frac{1}{8} = (\frac{1}{2})^{t/25}$ Simplify.

$(\frac{1}{2})^3 = (\frac{1}{2})^{t/25}$ $\frac{1}{8} = (\frac{1}{2})^3$

$3 = \frac{t}{25}$ If $b^x = b^y$, then $x = y$.

$75 = t$

Therefore, in year $1980 - 75$, or 1905, the population of the town was 6,400.

4. D

The table shows that one-half of the substance decays every 28 years. Therefore, the half-life of the radioactive substance is 28 years. Use the half-life formula, $A = P(\frac{1}{2})^{t/d}$, to find out how much of the original amount of the substance will remain after 140 years. P is the initial amount, t is the number of years and d is the half-life.

$A = 1,200(\frac{1}{2})^{140/28}$

$= 37.5$ Use a calculator.

To the nearest gram, 38 grams of the substance will remain after 140 years.

5. B

If the substance decays at a rate of 18% per year the amount of substance remaining each year will be multiplied by $(1 - 0.18)$, or 0.82.

The initial amount of 100 grams will become

$100(1 - 0.18)$ one year later,

$100(1 - 0.18)(1 - 0.18)$ two years later,

$100(1 - 0.18)(1 - 0.18)(1 - 0.18)$ three years later,

and so on. Thus, t years later, the remaining amount of the substance, in grams, is

$f(t) = 100(0.82)^t$.

Answers
Exponential Functions

6. C

$$5,000(1+\frac{r}{100})^t$$

The value of the 15 year investment at 6% annual compound interest

$$= 5,000(1+\frac{6}{100})^{15} = 5,000(1.06)^{15}.$$

The value of the 12 year investment at 6% annual compound interest

$$= 5,000(1+\frac{6}{100})^{12} = 5,000(1.06)^{12}.$$

The difference is

$$= 5,000(1.06)^{15} - 5,000(1.06)^{12}$$

$$= 5,000\left[(1.06)^{15} - (1.06)^{12}\right]$$

7. 8485

$$P(t) = 24,000(\frac{1}{2})^{\frac{t}{6}}$$

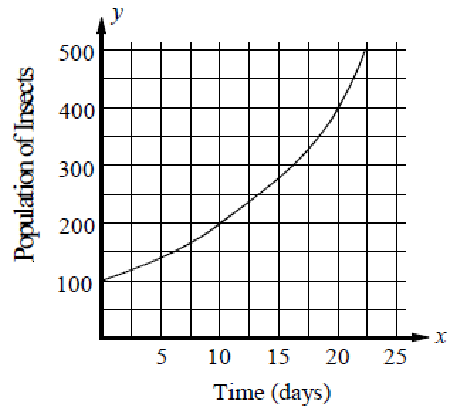
$$P(9) = 24,000(\frac{1}{2})^{\frac{9}{6}}$$ Substitute 9 for t.

$$= 24,000(\frac{1}{2})^{\frac{3}{2}}$$

$$\approx 8,485.28$$ Use a calculator.

To the nearest dollar, the price of the truck 9 years after it was purchased is $8,485.

8. 10



$$f(t) = 100(2)^{\frac{t}{d}}$$

In the equation, d represents the amount of time it takes to double the population. The graph shows that the population was 100 at $t = 0$, 200 at $t = 10$, and 400 at $t = 20$. Therefore, the value of doubling time d is 10 days.

9. 283

$$f(t) = 100(2)^{\frac{t}{d}}$$

$$f(15) = 100(2)^{\frac{15}{10}} = 100(2)^{1.5}$$

$$\approx 282.84$$ Use a calculator.

The population of the insect after 15 days was 283, to the nearest whole number.