Practice Test

Radical Functions

1

If the graph of $f(x) = 2x^3 + bx^2 + 4x - 4$ intersects the x-axis at $(\frac{1}{2}, 0)$, and $(-2, k)$ lies on the graph of f, what is the value of k ?

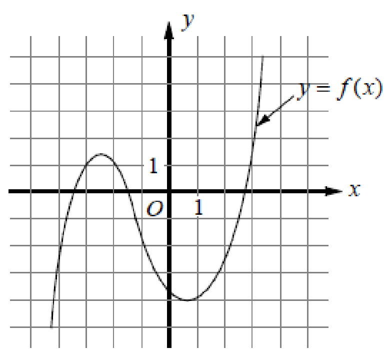
A) -4

B) -2

C) 0

D) 2

2



The function $y = f(x)$ is graphed on the xy-plane above. If k is a constant such that the equation $f(x) = k$ has one real solution, which of the following could be the value of k ?

A) -3

B) -1

C) 1

D) 3

3

What is the value of a if $x + 2$ is a factor of $f(x) = -(x^3 + 3x^2) - 4(x - a)$?

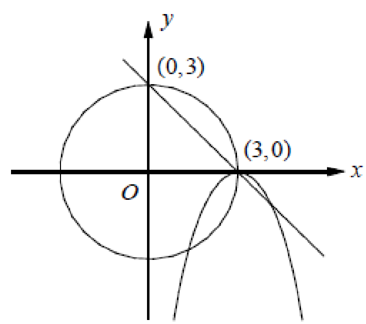
A) -2

B) -1

C) 0

D) 1

4



$x^2 + y^2 = 9$

$y = -(x - 3)^2$

$x + y = 3$

A system of three equations and their graphs on the xy-plane are shown above. How many solutions does the system have?

A) 1

B) 2

C) 3

D) 4

5

Which of the following complex numbers is equivalent to $\frac{(1-i)^2}{1+i}$?

A) $-\frac{i}{2}-\frac{1}{2}$

B) $-\frac{i}{2}+\frac{1}{2}$

C) $-i-1$

D) $-i+1$

6

Which of the following is equal to $a\sqrt[3]{a}$?

A) $a^{\frac{2}{3}}$

B) $a^{\frac{4}{3}}$

C) $a^{\frac{5}{3}}$

D) $a^{\frac{7}{3}}$

7

$$p(x) = -2x^3 + 4x^2 - 10x$$
$$q(x) = x^2 - 2x + 5$$

The polynomials $p(x)$ and $q(x)$ are defined above. Which of the following polynomials is divisible by $x-1$?

A) $f(x) = p(x) - \frac{1}{2}q(x)$

B) $g(x) = -\frac{1}{2}p(x) - q(x)$

C) $h(x) = -p(x) + \frac{1}{2}q(x)$

D) $k(x) = \frac{1}{2}p(x) + q(x)$

8

$$\sqrt{2x+6} = x+3$$

What is the solution set of the equation above?

A) $\{-3\}$

B) $\{-1\}$

C) $\{-3, 2\}$

D) $\{-3, -1\}$

9

What is the remainder when polynomial $p(x) = 24x^3 - 36x^2 + 14$ is divided by $x - \frac{1}{2}$?

A) 4

B) 6

C) 8

D) 10

10

The function f is defined by a polynomial. If $x+2$, $x+1$, and $x-1$ are factors of f, which of the following table could define f?

A)

x	$f(x)$
-2	4
-1	0
1	0
2	0

B)

x	$f(x)$
-2	0
-1	4
1	0
2	0

C)

x	$f(x)$
-2	0
-1	0
1	4
2	0

D)

x	$f(x)$
-2	0
-1	0
1	0
2	4

Answers
Radical Functions

1. C

$f(x) = 2x^3 + bx^2 + 4x - 4$

$f(\frac{1}{2}) = 0$ because the graph of f intersects the x-axis at $(\frac{1}{2}, 0)$.

$f(\frac{1}{2}) = 2(\frac{1}{2})^3 + b(\frac{1}{2})^2 + 4(\frac{1}{2}) - 4 = 0$

Solving the equation for b gives $b = 7$.

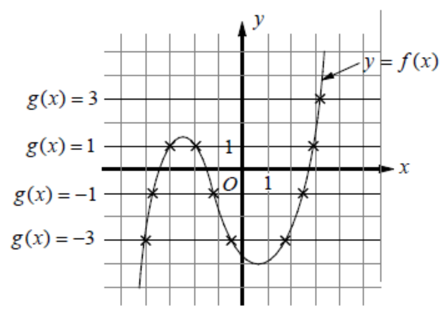
Thus $f(x) = 2x^3 + 7x^2 + 4x - 4$.

Also $k = f(-2)$, because $(-2, k)$ lies on the graph of f.

$k = f(-2) = 2(-2)^3 + 7(-2)^2 + 4(-2) - 4$

Solving the equation for k gives $k = 0$.

2. D



$g(x) = -3$ has 3 points of intersection with $y = f(x)$, so there are 3 real solutions.

$g(x) = -1$ has 3 points of intersection with $y = f(x)$, so there are 3 real solutions.

$g(x) = 1$ has 3 points of intersection with $y = f(x)$, so there are 3 real solutions.

$g(x) = 3$ has 1 point of intersection with $y = f(x)$, so there is 1 real solution.

Choice D is correct

3. B

If $x + 2$ is a factor of $f(x) = -(x^3 + 3x^2) - 4(x - a)$, then $f(-2) = 0$.

$f(-2) = -((-2)^3 + 3(-2)^2) - 4(-2 - a) = 0$

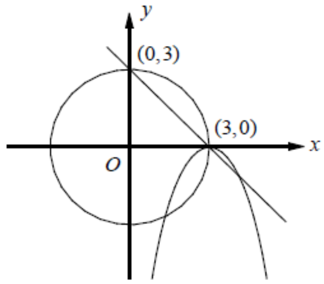
$-(-8 + 12) + 8 + 4a = 0$

$4 + 4a = 0$

$a = -1$

Answers
Radical Functions

4. A



The solutions to the system of equations are the points where the circle, parabola, and line all intersect. That point is $(3,0)$ and is therefore the only solution to the system.

5. C

$\frac{(1-i)^2}{1+i}$

$=\frac{1-2i+i^2}{1+i}$ FOIL the numerator.

$=\frac{1-2i-1}{1+i}$ $i^2=-1$

$=\frac{-2i}{1+i}$ Simplify.

$=\frac{-2i}{1+i}\cdot\frac{1-i}{1-i}$ Rationalize the denominator.

$=\frac{-2i+2i^2}{1-i^2}$ FOIL

$=\frac{-2i-2}{2}$ $i^2=-1$

$=-i-1$

6. B

$a\sqrt[3]{a}=a\cdot a^{\frac{1}{3}}=a^{1+\frac{1}{3}}=a^{\frac{4}{3}}$

7. B

$p(x)=-2x^3+4x^2-10x$

$q(x)=x^2-2x+5$

In $p(x)$, factoring out the GCF, $-2x$, yields

$p(x)=-2x(x^2-2x+5)=-2x\cdot q(x)$.

Let's check each answer choice.

A) $f(x)=p(x)-\frac{1}{2}q(x)$

$=-2x\cdot q(x)-\frac{1}{2}q(x)=(-2x-\frac{1}{2})q(x)$

$q(x)$ is not a factor of $x-1$ and $(-2x-\frac{1}{2})$ is not a factor of $x-1$. $f(x)$ is not divisible by $x-1$.

B) $g(x)=-\frac{1}{2}p(x)-q(x)$

$=-\frac{1}{2}[-2x\cdot q(x)]-q(x)=(x-1)q(x)$

Since $g(x)$ is $x-1$ times $q(x)$, $g(x)$ is divisible by $x-1$.

Choices C and D are incorrect because $x-1$ is not a factor of the polynomials $h(x)$ and $k(x)$.

8. D

$\sqrt{2x+6}=x+3$

$(\sqrt{2x+6})^2=(x+3)^2$ Square each side.

$2x+6=x^2+6x+9$ Simplify.

$x^2+4x+3=0$ Make one side 0.

$(x+1)(x+3)=0$ Factor.

$x+1=0$ or $x+3=0$ Zero Product Property

$x=-1$ or $x=-3$

Check each x-value in the original equation.

$\sqrt{2(-1)+6}=-1+3$ $x=-1$

$\sqrt{4}=2$ Simplify.

$2=2$ True

$\sqrt{2(-3)+6}=-3+3$ $x=-3$

$0=0$ True

Thus, -1 and -3 are both solutions to the equation.

9. C

Use the remainder theorem.

$p(\frac{1}{2})=24(\frac{1}{2})^3-36(\frac{1}{2})^2+14=8$

Therefore, the remainder of polynomial $p(x)=24x^3-36x^2+14$ divided by $x-\frac{1}{2}$ is 8.

10. D

If $(x-a)$ is a factor of $f(x)$, then $f(a)$ must equal to 0. Thus, if $x+2$, $x+1$ and $x-1$ are factors of f, we have $f(-2)=f(-1)=f(1)=0$.

Choice D is correct.