

Practice Test

Rational Functions

1

If $a \neq b$, which of the following is equivalent to $\frac{a}{a-b} + \frac{b}{b-a}$?

- A) 1
- B) $\frac{a+b}{a-b}$
- C) $\frac{a+b}{(a-b)^2}$
- D) $\frac{a^2+b^2}{(a-b)^2}$

2

If $x > 0$ and $y > 0$, which of the following is

equivalent to $\frac{\frac{1}{x} - \frac{1}{y}}{\frac{1}{x^2} - \frac{1}{y^2}}$?

- A) $\frac{xy}{x^2 - y^2}$
- B) $\frac{2xy}{x^2 - y^2}$
- C) $\frac{xy}{x+y}$
- D) $\frac{xy}{x-y}$

3

$$\frac{(k+1)^2}{k} = 4k$$

What is the solution set of the equation above?

- A) $\{-\frac{1}{3}\}$
- B) $\{-1\}$
- C) $\{-\frac{1}{3}, 1\}$
- D) $\{\frac{1}{3}, -1\}$

4

$$\frac{3}{x} - \frac{x}{x+2} = \frac{2}{x+2}$$

What is the solution set of the equation above?

- A) $\{2, -3\}$
- B) $\{-2, 3\}$
- C) $\{-2\}$
- D) $\{3\}$

5

$$\frac{x}{x+1} + \frac{4}{x-4} = \frac{20}{x^2 - 3x - 4}$$

What is the solution set of the equation above?

- A) $\{-4\}$
- B) $\{4\}$
- C) $\{-4, 4\}$
- D) There are no solutions to the equation.

6

If $x \neq \pm 1$, which of the following is equivalent

to $\frac{1 + \frac{1}{x-1}}{1 - \frac{1}{x+1}}$?

- A) $\frac{x-1}{x+1}$
 B) $\frac{x+1}{x-1}$
 C) $\frac{x^2-1}{x^2+1}$
 D) $\frac{x^2+1}{x^2-1}$

7

Working alone, Gary can load an empty truck in 3 hours. Working alone, his brother can load the same truck in x hours. If Gary and his brother worked together for t hours to load the empty truck, which of the following equations can be used to find out how much work was done during t hours?

- A) $\frac{3}{t} + xt$
 B) $\frac{3}{t} + \frac{x}{t}$
 C) $3t + xt$
 D) $\frac{1}{3}t + \frac{1}{x}t$

8

$$f(x) = \frac{5}{2(x-2)^2 - 3(x-2) - 2}$$

What is one possible value of x , if function f is undefined?

9

If $x > 0$, what is the solution to the equation

$$\frac{1}{2x} + \frac{3}{10x^2} = \frac{1}{5}?$$

10

If $a \neq b$ and $\frac{ab}{a-b} \div \frac{ab^2}{b-a} = -\frac{1}{6}$, what is the value of b ?

11

If $\frac{a + \frac{1}{2}}{a - \frac{1}{2}} = 2$, what is the value of a ?

Answers

Rational Functions

1. A

$$\frac{a}{a-b} + \frac{b}{b-a}$$

$$= \frac{a}{a-b} - \frac{b}{a-b}$$

$$= \frac{a-b}{a-b}$$

$$= 1$$

$$b - a = -(a - b)$$

Add the numerators.

2. C

$$\frac{\frac{1}{x} - \frac{1}{y}}{\frac{1}{x^2} - \frac{1}{y^2}}$$

Answers Rational Functions

Multiply x^2y^2 by the numerator and the denominator.

$$\begin{aligned} & \frac{\left(\frac{1}{x} - \frac{1}{y}\right)x^2y^2}{\left(\frac{1}{x^2} - \frac{1}{y^2}\right)x^2y^2} \\ &= \frac{xy^2 - x^2y}{y^2 - x^2} && \text{Distributive property} \\ &= \frac{xy \cancel{(y-x)}}{\cancel{(y-x)}(y+x)} \\ &= \frac{xy}{(y+x)} \end{aligned}$$

3. C

$$\begin{aligned} \frac{(k+1)^2}{k} &= 4k \\ (k+1)^2 &= 4k^2 && \text{Multiply by } k \text{ on each side.} \\ k^2 + 2k + 1 &= 4k^2 && \text{FOIL} \\ 0 &= 3k^2 - 2k - 1 && \text{Make one side 0.} \\ 0 &= (3k+1)(k-1) && \text{Factor.} \\ k &= -\frac{1}{3} \text{ or } k = 1 \end{aligned}$$

None of the solutions make the denominator zero, thus $\{-\frac{1}{3}, 1\}$ is the solution set.

Choice C is correct.

4. D

$$\begin{aligned} \frac{3}{x} - \frac{x}{x+2} &= \frac{2}{x+2} \\ \text{Multiply each side of the equation by } x(x+2). \\ x(x+2)\left(\frac{3}{x} - \frac{x}{x+2}\right) &= x(x+2)\left(\frac{2}{x+2}\right) \\ 3(x+2) - x^2 &= 2x && \text{Distributive property} \\ 3x + 6 - x^2 &= 2x && \text{Distributive property} \\ 0 &= x^2 - x - 6 && \text{Make one side 0.} \\ 0 &= (x+2)(x-3) && \text{Factor.} \\ x &= -2 \text{ or } x = 3 \end{aligned}$$

When x equals -2 , the denominator in the original equation has a value of 0. Therefore, -2 cannot be a solution.

The solution set is $\{3\}$.

5. A

$$\begin{aligned} \frac{x}{x+1} + \frac{4}{x-4} &= \frac{20}{x^2 - 3x - 4} \\ x^2 - 3x - 4 &= (x+1)(x-4). \text{ So the LCD is } \\ &(x+1)(x-4). \text{ Multiply each side of the equation} \\ &\text{by } (x+1)(x-4). \\ (x+1)(x-4)\left(\frac{x}{x+1} + \frac{4}{x-4}\right) & \\ &= (x+1)(x-4)\left(\frac{20}{x^2 - 3x - 4}\right) \\ x(x-4) + 4(x+1) &= 20 && \text{Distributive property} \\ x^2 - 4x + 4x + 4 &= 20 \\ x^2 &= 16 \\ x &= 4 \text{ or } x = -4 \end{aligned}$$

When x equals 4 , the denominator in the original equation has a value of 0. Therefore, 4 cannot be a solution.

The solution set is $\{-4\}$.

6. B

$$\begin{aligned} 1 + \frac{1}{x-1} \\ 1 - \frac{1}{x+1} \\ \frac{(x+1)(x-1)\left(1 + \frac{1}{x-1}\right)}{(x+1)(x-1)\left(1 - \frac{1}{x+1}\right)} &&& \text{Multiply by } (x+1)(x-1). \\ = \frac{(x+1)(x-1) + (x+1)}{(x+1)(x-1) - (x-1)} &&& \text{Distributive property} \\ = \frac{x^2 - 1 + x + 1}{x^2 - 1 - x + 1} &&& \text{FOIL} \\ = \frac{x^2 + x}{x^2 - x} &&& \text{Simplify.} \\ = \frac{x(x+1)}{x(x-1)} &&& \text{Factor.} \\ = \frac{x+1}{x-1} &&& \text{Cancel and simplify.} \end{aligned}$$

7. D

If working alone Gary can load the empty truck in 3 hours, his work rate is $\frac{1}{3}$. If working alone his brother can load the same truck in x hours, his work rate is $\frac{1}{x}$. If they work together for t

Answers Rational Functions

hours to load the empty truck, the amount of work done for t hours will be $t(\frac{1}{3} + \frac{1}{x})$, or $\frac{1}{3}t + \frac{1}{x}t$.

8. $\frac{3}{2}$ or 4

The expression $f(x) = \frac{5}{2(x-2)^2 - 3(x-2) - 2}$ is undefined when the denominator of $f(x)$ is zero. Therefore, if $2(x-2)^2 - 3(x-2) - 2$ is equal to 0, $f(x)$ is undefined.

$$2(x-2)^2 - 3(x-2) - 2 = 0$$

Let $z = x - 2$, then $2z^2 - 3z - 2 = 0$.

$$(2z+1)(z-2) = 0 \quad \text{Factor.}$$

$$2z+1=0 \text{ or } z-2=0 \quad \text{Zero Product Property}$$

$$z = -\frac{1}{2} \text{ or } z = 2$$

Now substitute $x-2$ for z .

$$x-2 = -\frac{1}{2} \text{ or } x-2 = 2$$

The values of x that make f undefined are

$$\frac{3}{2} \text{ and } 4.$$

9. 3

$$\frac{1}{2x} + \frac{3}{10x^2} = \frac{1}{5}$$

Multiply each side of the equation by $10x^2$.

$$10x^2 \left(\frac{1}{2x} + \frac{3}{10x^2} \right) = 10x^2 \left(\frac{1}{5} \right)$$

$$5x + 3 = 2x^2 \quad \text{Distributive property}$$

$$0 = 2x^2 - 5x - 3 \quad \text{Make one side 0.}$$

$$0 = (2x+1)(x-3) \quad \text{Factor.}$$

$$x = -\frac{1}{2} \text{ or } x = 3$$

Since $x > 0$, the only solution is 3.

10. 6

$$\frac{ab}{a-b} \div \frac{ab^2}{b-a} = -\frac{1}{6}$$

Rewrite as multiplication.

$$\frac{ab}{a-b} \times \frac{b-a}{ab^2} = -\frac{1}{6}$$

$$\frac{\cancel{a}\cancel{b}}{\cancel{a}\cancel{b}} \times \frac{-(\cancel{a}\cancel{b})}{\cancel{a}\cancel{b}^2} = -\frac{1}{6} \quad b-a = -(a-b)$$

$$\frac{-1}{b} = -\frac{1}{6}$$

Therefore, the value of b is 6.

11. $\frac{3}{2}$

$$\frac{a + \frac{1}{2}}{a - \frac{1}{2}} = 2$$

Multiply each side of the equation by $a - \frac{1}{2}$.

$$a + \frac{1}{2} = 2\left(a - \frac{1}{2}\right)$$

$$a + \frac{1}{2} = 2a - 1 \quad \text{Distributive property}$$

$$\frac{3}{2} = a$$