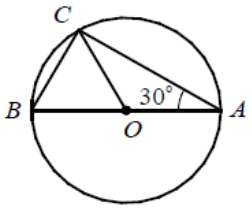
Practice Test

Circles

1



In the figure above, O is the center of the circle and $\overline{AB}$ is a diameter. If the length of $\overline{AC}$ is $4\sqrt{3}$ and $m\angle BAC = 30$, what is the area of circle O?

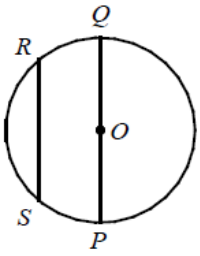
A) 12π

B) 16π

C) 18π

D) 24π

2



In the circle above, chord $\overline{RS}$ is parallel to diameter $\overline{PQ}$. If the length of $\overline{RS}$ is $\frac{3}{4}$ of the length of $\overline{PQ}$ and the distance between the chord and the diameter is $2\sqrt{7}$, what is the radius of the circle?

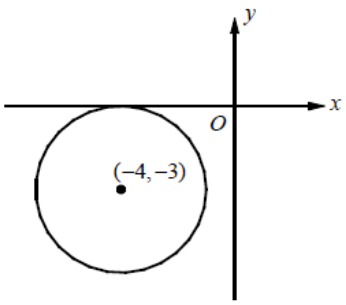
A) 6

B) $3\sqrt{7}$

C) 8

D) $4\sqrt{7}$

3



In the figure above, the circle is tangent to the x-axis and has center $(-4,-3)$. Which of the following equations represents the equation of the circle shown in the xy-plane above?

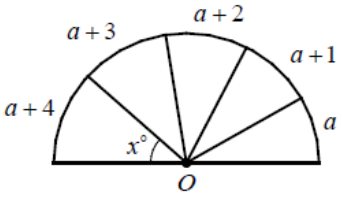
A) $(x+4)^2+(y+3)^2=9$

B) $(x-4)^2+(y-3)^2=9$

C) $(x+4)^2+(y+3)^2=3$

D) $(x-4)^2+(y-3)^2=3$

4



The figure above shows a semicircle with the lengths of the adjacent arcs a, $a+1$, $a+2$, $a+3$, and $a+4$. If the value of x is 42, what is the value of a?

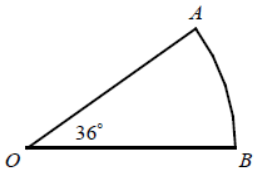
A) 7

B) 8

C) 9

D) 10

5



In the figure above, the length of arc $\overset{\frown}{AB}$ is π. What is the area of sector OAB?

A) 2π

B) $\frac{5}{2}\pi$

C) 3π

D) $\frac{7}{2}\pi$

6

$$x^2 - 4x + y^2 - 6x - 17 = 0$$

What is the area of the circle in the xy- plane above?

A) 20π

B) 24π

C) 26π

D) 30π

7

Which of the following is the equation of a circle that has a diameter of 8 units and is tangent to the graph of $y = 2$?

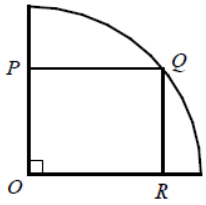
A) $(x+1)^2 + (y+2)^2 = 16$

B) $(x-1)^2 + (y-2)^2 = 16$

C) $(x+2)^2 + (y+1)^2 = 16$

D) $(x-2)^2 + (y-1)^2 = 16$

8



In the figure above, rectangle $OPQR$ is inscribed in a quarter circle that has a radius of 9. If $PQ = 7$ what is the area of rectangle $OPQR$?

A) $24\sqrt{2}$

B) $26\sqrt{2}$

C) $28\sqrt{2}$

D) $30\sqrt{2}$

9

In a circle with center O, the central angle has a measure of $\frac{2\pi}{3}$ radians. The area of the sector formed by central angle AOB is what fraction of the area of the circle?

10

A wheel with a radius of 2.2 feet is turning at a constant rate of 400 revolutions per minute on a road. If the wheel traveled $k\pi$ miles in one hour what is the value of k? (1 mile = 5,280 feet)

Answers
Circles

1. B

An angle inscribed in a semicircle is a right angle. Therefore, $\angle ACB = 90$.

So, ΔABC is a $30°$-$60°$-$90°$ triangle.

In a $30°$-$60°$-$90°$ triangle, the hypotenuse is twice as long as the shorter leg and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

$AC = \sqrt{3}BC$

$4\sqrt{3} = \sqrt{3}BC$ $\qquad AC = 4\sqrt{3}$

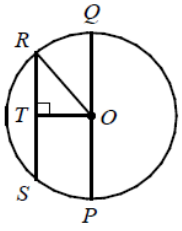
$4 = BC$

$AB = 2BC = 2(4) = 8$

Therefore, the radius of circle O is 4.

Area of circle $O = \pi(4)^2 = 16\pi$

2. C



Draw $\overline{OR}$ and $\overline{OT}$ as shown above. Let the radius of the circle be r, then $OQ = OR = r$.

Since the ratio of RS to QP is 3 to 4, the ratio of RT to OQ is also 3 to 4.

Therefore, $RT = \frac{3}{4}OQ = \frac{3}{4}r$.

OT is the distance between the chord and the diameter, which is given as $2\sqrt{7}$.

$OR^2 = RT^2 + OT^2$ Pythagorean Theorem

$r^2 = (\frac{3}{4}r)^2 + (2\sqrt{7})^2$ Substitution

$r^2 = \frac{9}{16}r^2 + 28$ Simplify.

$r^2 - \frac{9}{16}r^2 = 28$

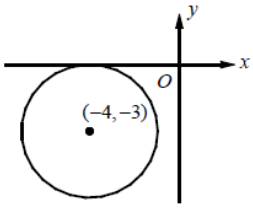
$\frac{7}{16}r^2 = 28$

$\frac{16}{7}\cdot\frac{7}{16}r^2 = \frac{16}{7}\cdot 28$

$r^2 = 64$

$r = \sqrt{64} = 8$

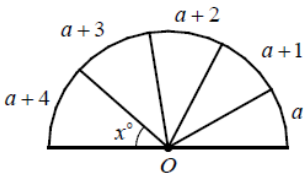
3. A



If the center of the circle is $(-4,-3)$ and the circle is tangent to the x-axis, the radius is 3.

The equation is $(x-(-4))^2 + (y-(-3))^2 = 3^2$, or $(x+4)^2 + (y+3)^2 = 9$.

4. D



The arc length of the semicircle is $(a+4)+(a+3)+(a+2)+(a+1)+a = 5a+10$.

In a circle, the lengths of the arcs are proportional to the degree measures of the corresponding arcs.

Therefore, $\frac{\text{arc length of semicircle}}{180°} = \frac{a+4}{x°}$.

$\frac{5a+10}{180} = \frac{a+4}{42}$ Substitution

$42(5a+10) = 180(a+4)$ Cross Products

$210a + 420 = 180a + 720$

$30a = 300$

$a = 10$

Answers
Circles

5. B

Length of arc $AB = 2\pi r \cdot \frac{m\angle AOB}{360}$

$= 2\pi r \cdot \frac{36}{360} = \frac{\pi r}{5}$

Since the length of the arc is given as π,

$\frac{\pi r}{5} = \pi$. Solving the equation for r gives $r = 5$.

Area of sector $AOB = \pi r^2 \cdot \frac{m\angle AOB}{360}$

$= \pi(5)^2 \cdot \frac{36}{360} = \frac{5}{2}\pi$

6. D

$x^2 - 4x + y^2 - 6x - 17 = 0$

$x^2 - 4x + y^2 - 6x = 17$

To complete the square, add $(-4 \cdot \frac{1}{2})^2 = 4$ and

$(-6 \cdot \frac{1}{2})^2 = 9$ to each side.

$x^2 - 4x + 4 + y^2 - 6x + 9 = 17 + 4 + 9$

$(x-2)^2 + (y-3)^2 = 30$

The radius of the circle is $\sqrt{30}$, the area of the circle is $\pi(\sqrt{30})^2 = 30\pi$

7. A

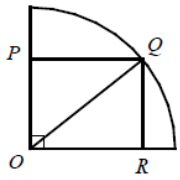
If the diameter of the circle is 8 units, the radius of the circle is 4 units. Since the radius of the circle is 4 units, the y- coordinate of the center has to be 4 units above or below $y = 2$.

The y- coordinate of the center has to be either 6 or -2. Among the answer choices, only choice A has -2 as the y- coordinate.

No other answer choice has 6 or -2 as the y- coordinate of the center.

Choice A is correct.

8. C



Draw $\overline{OQ}$. Since $\overline{OQ}$ is a radius, $OQ = 9$.

$OP^2 + PQ^2 = OQ^2$ Pythagorean Theorem

$OP^2 + 7^2 = 9^2$ Substitution

$OP^2 = 9^2 - 7^2 = 32$

$OP = \sqrt{32} = \sqrt{16} \cdot \sqrt{2} = 4\sqrt{2}$

Area of rectangle $OPQR = OP \times PQ$

$= 4\sqrt{2} \times 7 = 28\sqrt{2}$

9. $\frac{1}{3}$

Area of sector $AOB = \pi r^2 \cdot \frac{m\angle AOB}{360}$

The area of a sector is the fractional part of the area of a circle. The area of a sector formed by $\frac{2\pi}{3}$ radians of arc is $\frac{2\pi/3}{2\pi}$, or $\frac{1}{3}$, of the area of the circle.

10. 20

The distance the wheel travels in 1 minute is equal to the product of the circumference of the wheel and the number of revolutions per minute.

The distance the wheel travels in 1 minute

$= 2\pi r \times$ the number of revolutions per minute

$= 2\pi(2.2 \text{ ft}) \times 400 = 1,760\pi \text{ ft}$

Total distance traveled in 1 hour

$= 1,760\pi \text{ ft} \times 60 = 105,600\pi \text{ ft}$

$= 105,600\pi \text{ ft} \times \frac{1 \text{ mile}}{5,280 \text{ ft}} = 20\pi \text{ miles}$

Thus, $k = 20$.