SAT: Algebra and Functions

Algebra and Functions

1. Let m be an even integer. How many possible values of m satisfy $\sqrt{m+7} \leq 3$?

(A) One
(B) Two
(C) Three
(D) Four
(E) Five

2. Let $\boxed{x}$ be defined by $\boxed{x} = \dfrac{x+3}{x-1}$ for any x such that $x \neq 1$. Which of the following is equivalent to $\boxed{x} - 1$?

(A) $\dfrac{x+2}{x-1}$

(B) $\dfrac{4}{x-1}$

(C) $\dfrac{2x+4}{x-1}$

(D) $\dfrac{2}{x-1}$

(E) $\dfrac{x+2}{x-2}$

3. Let a and b be numbers such that $a^3 = b^2$. Which of the following is equivalent to $b\sqrt{a}$?

(A) $b^{2/3}$

(B) $b^{4/3}$

(C) b^2

(D) b^3

(E) b^4

4. Let m and n be positive integers such that one-third of m is n less than one-half of m. Which of the following is a possible value of m?

(A) 15
(B) 21
(C) 24
(D) 26
(E) 28

5. If a and b are numbers such that $(a-4)(b+6) = 0$, then what is the smallest possible value of $a^2 + b^2$?

6. Let $f(x) = ax^2$ and $g(x) = bx^4$ for any value of x. If a and b are positive constants, for how many values of x is $f(x) = g(x)$?

(A) None
(B) One
(C) Two
(D) Three
(E) Four

7. Let a and b be numbers such that $30 < a < 40$ and $50 < b < 70$. Which of the following represents all possible values of $a - b$?

(A) $-40 < a-b < -20$
(B) $-40 < a-b < -10$
(C) $-30 < a-b < -20$
(D) $-20 < a-b < -10$
(E) $-20 < a-b < 30$

Answers

Algebra and Functions

1. E (Estimated Difficulty Level: 5)

The answers suggest that there aren't that many possibilities. So, make up some even integers, plug them in for m, and see if they work! Since we can't take the square root of a negative number, m can't be less than -6. Also, if $m = 2$, then $\sqrt{m+7} = 3$, but any larger value of m won't work. So, the possible values are -6, -4, -2, 0, and 2. (Don't forget that zero is a perfectly good even integer.)

2. B (Estimated Difficulty Level: 4)

Plug in real numbers for x! You can plug in anything other than 1. If you set $x = 2$, then $\boxed{x} - 1 = 5 - 1 = 4$. Now go through the answers, plugging in 2 for x. You will find that answers A and B are both equal in value to 4. If this happens, simply plug in another number. (You don't need to retry the answers that were wrong.) If $x = 0$, then $\boxed{x} - 1 = -3 - 1 = -4$, and only answer B is also -4, so that is the correct answer.

If you love algebra, here is how to do it:

$$\begin{aligned}\boxed{x} - 1 &= \frac{x+3}{x-1} - 1\\ &= \frac{x+3}{x-1} - \frac{x-1}{x-1}\\ &= \frac{x+3-(x-1)}{x-1}\\ &= \frac{4}{x-1}\end{aligned}$$

Be still my beating heart!

3. B (Estimated Difficulty Level: 5)

Solve the first equation given for a: $a = b^{2/3}$. Then $\sqrt{a} = a^{1/2} = b^{1/3}$. (You really need to know your exponent rules for this one.) So, $b\sqrt{a} = b \cdot b^{1/3} = b^{4/3}$. Using real numbers also works here, but it may be hard to come up with two that work for a and b (such as $a = 4$ and $b = 8$).

4. C (Estimated Difficulty Level: 4)

Translate the words into an algebraic equation:

$$\frac{m}{3} = \frac{m}{2} - n.$$

Multiplying both sides by 6 (the common denominator) gives $2m = 3m - 6n$, or $m = 6n$. So, m must be a positive multiple of 6, which means that answer C is correct.

5. 16 (Estimated Difficulty Level: 4)

Since $(a-4)(b+6) = 0$, the possible solutions are: $a = 4$ and b is anything, or $b = -6$ and a is anything. Now, the expression $a^2 + b^2$ is made smallest by choosing a and b to be close to zero as possible. So, $a = 4$ and $b = 0$ will give us the smallest value of $a^2 + b^2$, namely, 16. Using the other solution would give $a^2 + 36$, which will always be bigger than 16.

6. D (Estimated Difficulty Level: 5)

This is a tough one. For $f(x)$ to be equal to $g(x)$ for all x, we need $ax^2 = bx^4$. First, notice that if $x = 0$, both sides are zero, so $x = 0$ is a solution. If x is not zero, we can divide both sides of the equation by x^2 to get: $a = bx^2$. Solving for x results in $x = \pm\sqrt{a/b}$. This makes three solutions total, so answer D is correct. It may help to plug in numbers for a and b to make this problem more concrete.

7. B (Estimated Difficulty Level: 5)

To make $a - b$ as large as possible, we need to make a as large as possible and b as small as possible. So, $a - b$ has to be less than $40 - 50 = -10$. To make $a - b$ as small as possible, we need to make a as small as possible and b as large as possible. So, $a - b$ has to be greater than $30 - 70 = -40$. The expression that gives all possible values of $a - b$ is then $-40 < a - b < -10$.

Algebra and Functions

$$\frac{x}{3}+\frac{y}{12}=z$$

8. In the equation shown above, x, y, and z are positive integers. All of the following could be a possible value of y EXCEPT

(A) 4
(B) 6
(C) 8
(D) 12
(E) 20

$$\sqrt{72}+\sqrt{72}=m\sqrt{n}$$

9. In the equation above, m and n are integers such that $m > n$. Which of the following is the value of m?

(A) 6
(B) 12
(C) 16
(D) 24
(E) 48

t	0	1	2
$N(t)$	128	16	2

10. The table above shows some values for the function N. If $N(t) = k \cdot 2^{-at}$ for positive constants k and a, what is the value of a?

(A) -3

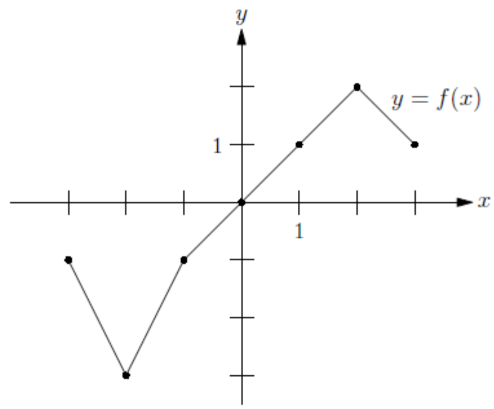
(B) -2

(C) $\frac{1}{3}$

(D) 2

(E) 3

11. Amy is two years older than Bill. The square of Amy's age in years is 36 greater than the square of Bill's age in years. What is the sum of Amy's age and Bill's age in years?



12. The function f is graphed in its entirety above. If the function g is defined so that $g(x) = f(-x)$, then for what value of x does g attain its maximum value?

(A) -3
(B) -2
(C) 0
(D) 2
(E) 3

13. If $(x+1)^2 = 4$ and $(x-1)^2 = 16$, what is the value of x?

(A) -3
(B) -1
(C) 1
(D) 3
(E) 5

Answers

Algebra and Functions

8. B (Estimated Difficulty Level: 5)

Solve the equation for y. You should get: $y = 12z - 4x$. Factor out a 4 from the right-hand side: $y = 4(3z - x)$. Since z and x are integers, $3z - x$ is an integer, so that y is a multiple of 4. Only answer B is not a multiple of 4, so it can't be a possible value of y.

You could also combine the fractions on the left-hand side of the equation to get:

$$\frac{4x + y}{12} = z.$$

For z to be an integer, $4x + y$ must be a multiple of 12. Try plugging in various integers for x and y to get multiples of 12; you should find that y can take on all of the values in the answers except for 6.

9. B (Estimated Difficulty Level: 4)

Combining the two terms on the left-hand side of the equation gives us $2\sqrt{72}$, but that doesn't give us what we need, which is $m > n$.

For the SAT test, you should know how to rewrite and simplify radicals. Here, $\sqrt{72} = \sqrt{36 \cdot 2} = 6\sqrt{2}$, so the left-hand side is equal to $12\sqrt{2}$, making $m = 12$ and $n = 2$.

10. E (Estimated Difficulty Level: 5)

Hint: if you see a zero in a table problem like this one, try to use it first! When you plug in 0 for t, you get $N(0) = k \cdot 2^{-0} = k \cdot 1 = k$, which means that $k = 128$.

Next, try plugging in 1 for t: $N(1) = 128 \cdot 2^{-a}$. From the table, $N(1) = 16$, so that $128 \cdot 2^{-a} = 16$, or $2^{-a} = 1/2^a = 1/8$. Since $8 = 2^3$, $a = 3$. (Your calculator may also help here, but try to understand how to do it without it.)

11. 18 (Estimated Difficulty Level: 4)

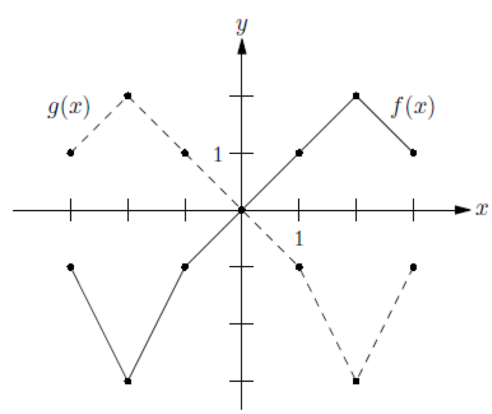
Let a be Amy's age and b be Bill's age. The problem tells us that: $a = b + 2$ and $a^2 = b^2 + 36$. One way to do this is to plow ahead, substitute for one variable and solve for the other (a bit messy!). But SAT questions are designed to be solved without tedious calculations and/or messy algebra. Let's try doing the problem using the SAT way, not the math teacher way.

First, notice that the second equation can be written as: $a^2 - b^2 = 36$. This is a difference of two squares, and is the same as: $(a+b)(a-b) = 36$. The first equation can be written as: $a - b = 2$. This means that the second equation is just $(a+b) \cdot 2 = 36$, so that $a + b = 18$. We don't know what a and b are, and we don't even care!

12. B (Estimated Difficulty Level: 4)

You can obtain the graph of $y = f(-x)$ by "flipping" the graph of $y = f(x)$ across the y-axis. For example, if the point $(3, 1)$ is on the graph of $f(x)$, then the point $(-3, 1)$ must be on the graph of $f(-x)$, since $f(-(-3)) = f(3) = 1$.

The figure below tells the story. Here, the function $g(x) = f(-x)$ is shown as a dashed line:



From the graph, $g(x)$ is maximum when $x = -2$.

Answers

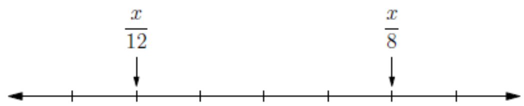
Algebra and Functions

13. A (Estimated Difficulty Level: 4)

A good "skip-the-algebra" way to do this problem is to use the answers by plugging them into x until the two given equations work. Using answer A, you should find that $(-3+1)^2 = (-2)^2 = 4$ and $(-3-1)^2 = (-4)^2 = 16$, so answer A is correct.

You must have the algebraic solution, you say? Try taking the square root of both sides of the equations, but don't forget that there are two possible solutions when you do this. The first equation gives: $x + 1 = \pm 2$ so that $x = 1$ or $x = -3$. The second equation gives $x - 1 = \pm 4$ so that $x = 5$ or $x = -3$. The only solution that works for both equations is $x = -3$.

Algebra and Functions



14. On the number line above, the tick marks correspond to consecutive integers. What is the value of x?

15. The value of y increased by 12 is directly proportional to the value of x decreased by 6. If $y = 2$ when $x = 8$, what is the value of x when $y = 16$?

(A) 8
(B) 10
(C) 16
(D) 20
(E) 28

16. Two cars are racing at a constant speed around a circular racetrack. Car A requires 15 seconds to travel once around the racetrack, and car B requires 25 seconds to travel once around the racetrack. If car A passes car B, how many seconds will elapse before car A once again passes car B?

Answers

Algebra and Functions

14. 96 (Estimated Difficulty Level: 5)

Since the tick marks correspond to consecutive integers, and it takes four "steps" to go from $x/12$ to $x/8$, we know that $x/8$ is four greater than $x/12$. (Or, think of the spaces between the tick marks: there are four spaces and each space is length 1, so the distance from $x/12$ to $x/8$ is 4.) In equation form:

$$\frac{x}{8} = \frac{x}{12} + 4.$$

Multiplying both sides by 24 gives: $3x = 2x + 4 \cdot 24$ so that $x = 96$.

15. B (Estimated Difficulty Level: 5)

First, recall that if y is proportional to x, then $y = kx$ for some constant k. So, "y increased by 12 is directly proportional to x decreased by 6" translates into the math equation: $y + 12 = k(x - 6)$. Plugging in $y = 2$ and $x = 8$ gives $14 = k \cdot 2$ so that $k = 7$. Our equation is now: $y + 12 = 7(x - 6)$. Plugging in 16 for y gives $28 = 7(x - 6)$ so that $x - 6 = 4$, or $x = 10$.

16. 75/2 or 37.5 (Estimated Difficulty Level: 5)

To make this problem more concrete, make up a number for the circumference of the racetrack. It doesn't really matter what number you use; I'll use 75 feet. Since speed is distance divided by time, the speed of car A is $75/15 = 5$ feet per second, and the speed of car B is $75/25 = 3$ feet per second. (I picked 75 mostly because it is divided evenly by 15 and 25.) Every second, car A gains 2 feet on car B. To pass car B, car A must gain 75 feet on car B. This will require $75/2 = 37.5$ seconds.

You may be thinking, "Whoa, tricky solution!" Here is the mostly straightforward but somewhat tedious algebraic solution. Once again, I'll use 75 feet for the circumference of the track. Suppose that you count time from when car A first passes car B. Then, car A travels a distance $(75/15)t = 5t$ feet after t seconds. (Remember that distance = speed × time.) For example, after 15 seconds, car A has traveled a distance $5 \cdot 15 = 75$ feet, and after 30 seconds, car A has traveled a distance $5 \cdot 30 = 150$ feet. Similarly, car B travels a distance $(75/25)t = 3t$ feet after t seconds. When the two cars pass again, car A has traveled 75 feet more than car B: $5t = 3t + 75$. Solving for t gives: $2t = 75$, or $t = 75/2 = 37.5$ seconds.