SAT: Data, Statistics, and Probability

Data, Statistics, and Probability

1. The first term of a sequence is the number n, and each term thereafter is 5 greater than the term before. Which of the following is the average (arithmetic mean) of the first nine terms of this sequence?

   (A) $n + 20$
   (B) $n + 180$
   (C) $2n$
   (D) $2n + 40$
   (E) $9n + 180$

2. The average (arithmetic mean) of a particular set of seven numbers is 12. When one of the numbers is replaced by the number 6, the average of the set increases to 15. What is the number that was replaced?

   (A) -20
   (B) -15
   (C) -12
   (D) 0
   (E) 12

3. Let a, b, and c be positive integers. If the average (arithmetic mean) of a, b, and c is 100, which of the following is NOT a possible value of any of the integers?

   (A) 1
   (B) 100
   (C) 297
   (D) 298
   (E) 299

4. M is a set consisting of a finite number of consecutive integers. If the median of the numbers in set M is equal to one of the numbers in set M, which of the following must be true?

   I. The average (arithmetic mean) of the numbers in set M equals the median.

   II. The number of numbers in set M is odd.

   III. The sum of the smallest number and the largest number in set M is even.

   (A) I only
   (B) II only
   (C) I and II only
   (D) I and III only
   (E) I, II, and III

SAT: Data, Statistics, and Probability

Answers

Data, Statistics, and Probability

1. A (Estimated Difficulty Level: 4)

The first nine terms of the sequence are:

$$n, \; n+5, \; n+10, \; n+15, \; \ldots, \; n+40.$$

(You should probably write all nine terms out to avoid mistakes.) Adding these terms up gives: $9n+180$. The average is the sum $(9n+180)$ divided by the number of terms (9). The average is then: $(9n+180)/9 = n+20$.

2. B (Estimated Difficulty Level: 5)

The average of a set of numbers is the sum of the numbers divided by the number of numbers:

$$\text{average} = \frac{\text{sum}}{N}.$$

We can solve this equation for the sum:

$$\text{sum} = \text{average} \times N.$$

Here, since there are 7 numbers and the average is 12, the sum of the numbers is $7 \times 12 = 84$. The sum of the new set of numbers is $7 \times 15 = 105$. Now, suppose that the seven numbers are a, b, c, d, e, f, and g, and that g gets replaced with the number 6. Then, we have:

$$a+b+c+d+e+f+g = 84,$$

and

$$a+b+c+d+e+f+6 = 105.$$

The second equation says that $a+b+c+d+e+f = 99$. Substituting into the first equation gives $99+g = 84$ so that $g = -15$.

3. E (Estimated Difficulty Level: 4)

Using the definition of average gives:

$$\frac{a+b+c}{3} = 100$$

so that $a+b+c = 300$. Since a, b, and c are all positive, the smallest possible value for any of the numbers is 1. The largest possible value of one of the three numbers then occurs when the other two numbers are both 1. In this case, the numbers are 1, 1, and 298, so that the largest possible value is 298. Answer E can not be a possible value, so it is the correct answer.

4. E (Estimated Difficulty Level: 5)

Plug in real numbers for set M to make this problem concrete. For example, if M is the set of consecutive integers from 1 to 5, then the median and average are both 3. If M is the set of consecutive integers from 1 to 4, then the median and average are both 2.5. From these examples, we can see that the number of numbers in set M needs to odd, otherwise the median is not an integer. Choice II must be true.

Also, if the number of numbers in a set of consecutive integers is odd, then when the first number is odd, the last number is odd. Or, when the first number is even, the last number is even. This is because the difference of the largest number and the smallest number will be even when the number of numbers is odd. Choice III must then be true, since the sum of two odd numbers or two even numbers is an even number.

At this point, the only answer with choices II and III is answer E, so that must be the correct answer. Why is choice I also correct? The average of a set of consecutive integers is equal to the average of the first and the last integers in the set. The average of two integers that are both odd or both even is the integer halfway between the two, which is also the median of the set. Whew!