Sum-to-Product Product-to-Sum

Product to Sum Formulas

$$\begin{split} & \sin(\alpha)\sin(\beta) = \frac{1}{2}\left[\cos(\alpha - \beta) - \cos(\alpha + \beta)\right] \\ & \cos(\alpha)\cos(\beta) = \frac{1}{2}\left[\cos(\alpha - \beta) + \cos(\alpha + \beta)\right] \\ & \sin(\alpha)\cos(\beta) = \frac{1}{2}\left[\sin(\alpha + \beta) + \sin(\alpha - \beta)\right] \\ & \cos(\alpha)\sin(\beta) = \frac{1}{2}\left[\sin(\alpha + \beta) - \sin(\alpha - \beta)\right] \end{split}$$

Sum to Product Formulas

$$\sin(\alpha) + \sin(\beta) = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$
$$\sin(\alpha) - \sin(\beta) = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$
$$\cos(\alpha) + \cos(\beta) = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$
$$\cos(\alpha) - \cos(\beta) = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$